அனைவருக்கும் இனிய காலை வணக்கம்.....

Electrostatics……. நிலைமின்னியல்……..

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Two main goals of physics

- Explore the new type of natural forces exist in the universe

- Explain the effects of natural forces on objects….

Before 18th century……

- Gravitational force…..
- Tension…
- Friction….
- Spring force…

- Effects of these forces on matter were clearly studied….

Charge.......

- Fundamental property like mass
- Every elementary particle has this property.
- Conservation of charge – Net change of charge is zero in any physical process

$$n \rightarrow p + e^{-} + \bar{\nu} \qquad e^{+} + e^{-} \rightarrow \gamma + \gamma$$

- Quantized $(q = ne)$ where $n = 0,1,2,3 \ldots$

  $e = 1.6 \times 10^{-19} C$

  significant in micro world

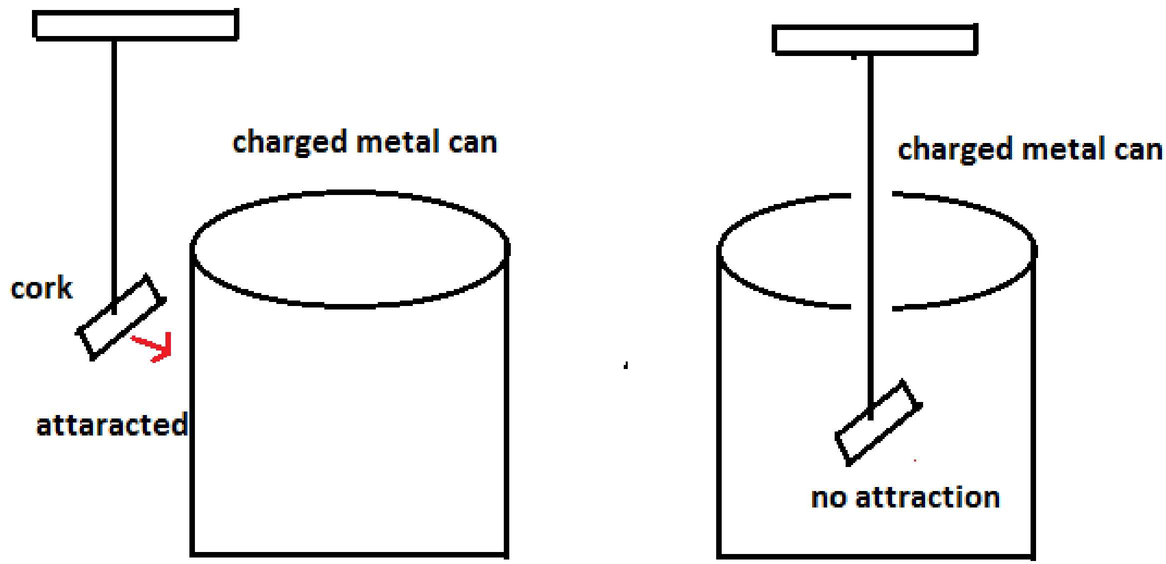
- Invariance of e: The value of the charge is independent of velocity. It has profound significance – Neutrality of atoms…
- Charge (property)–மின்னூட்டம்
- Charge (particle) –மின்துகள்

Coulomb's law

- Coincidentally Coulomb discovered his famous law…….

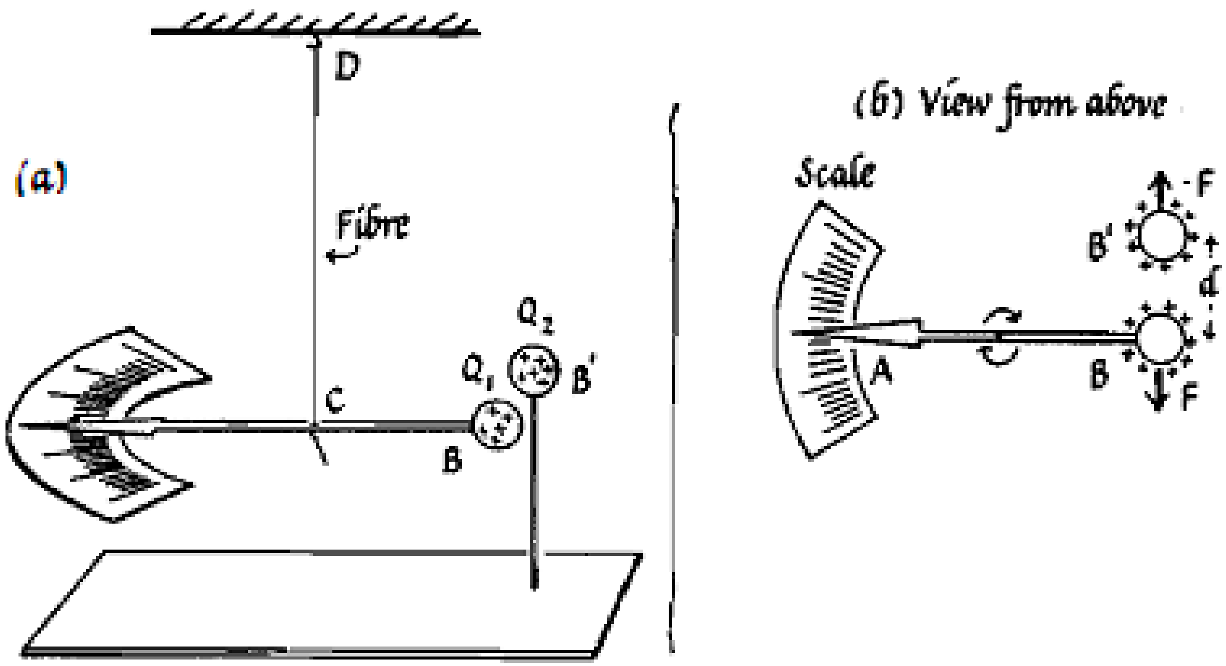
- JUNE,1785……

Benjamin Franklin (1775)



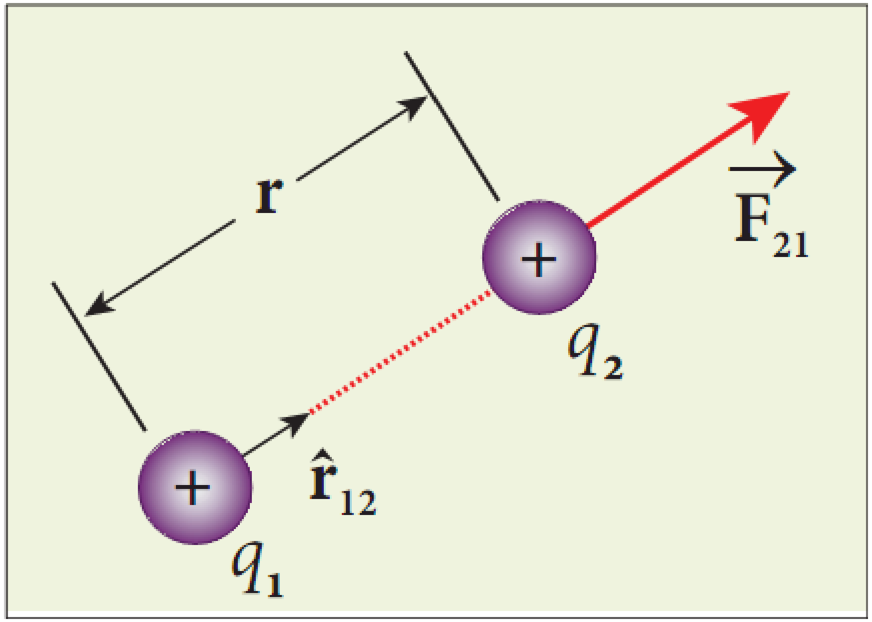
- James priestly (oxygen discoverer) – $F \propto \frac{1}{R^2}$
- He got intuition from Newton's result

Coulomb's apparatus…Torsional balance…



Coulomb's law……

$$\vec{F}_{21} = k\frac{q_1 q_2}{r^2}\hat{r}_{12} \qquad (1.2)$$



Important aspects…

- Product of two charges and inverse square of the distance

- Coulomb force lies along the line joining between two charges….

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

- Force between two charges depends on the medium in which these two charges are kept…

Gravitational force Vs Coulomb force

- Both are Inverse square law
- Both are central forces…..conservative forces
- Product of masses – product of charges
- Attractive – attractive , repulsive
- $G(6.67 \times 10^{-11}) <<<<< k(9 \times 10^{9})$
- Independent of motion – $(v = 0, Coulomb\ force$

  $(v \neq 0, Lorentz\ force)$

  - obeys Newton's third law

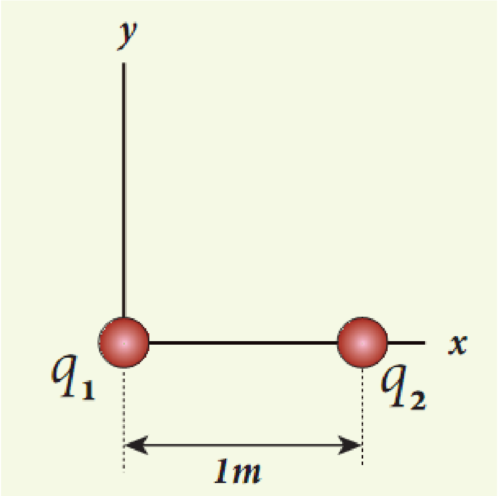
"... Coulomb's law. Like charges repel, and unlike charges attract each other, with a force that varies inversely as the square of the distance between them ... in *all* of atomic and molecular physics, in *all* solids, liquids, and gases, and in *all* things that involve our relationship with our environment, the *only* force law, besides gravity, is some manifestation of this simple law. Frictional forces, wind forces, chemical bonds, viscosity, magnetism, the forces that make the wheels of industry go round—all these are nothing but Coulomb's law ..."

—J. R. Zacharias

In *Science*, March 8. 1957.

Simple example

- $\vec{F}_{21} = k\frac{q_1 q_2}{r^2}\hat{r}_{12}$. (For simplicity k=1)



(1) If $q_1 = +1C$, $q_2 = +1C$ and $r = 1m$

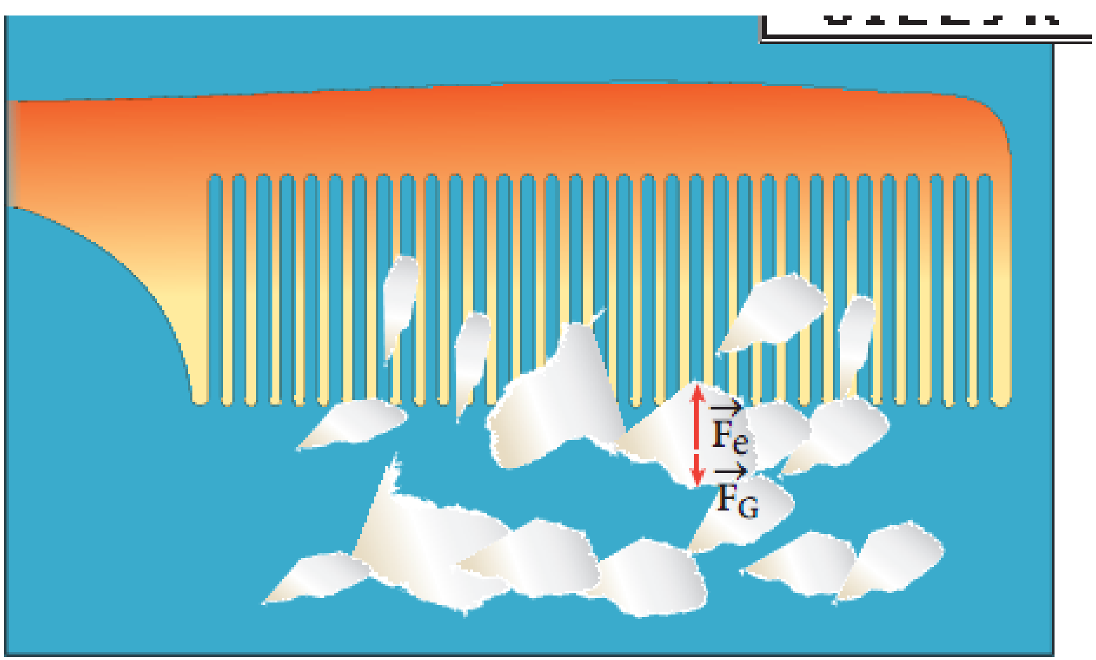
$$\vec{F}_{21} = \frac{(+1)(+1)}{1^2}\hat{\imath} = \hat{\imath} \qquad \text{(repulsive)}$$

(2) If $q_1 = +1C$, $q_2 = -1C$ and $r = 1m$

$$\vec{F}_{21} = \frac{(+1)(-1)}{1^2}\hat{\imath} = -\hat{\imath} \qquad \text{(attractive)}$$

Both cases $\hat{r}_{12} = \hat{\imath}$. But the minus comes from the charge. In general, the minus sign can come from either charge or unit vector

$$F_e \approx 10^{39} F_G$$

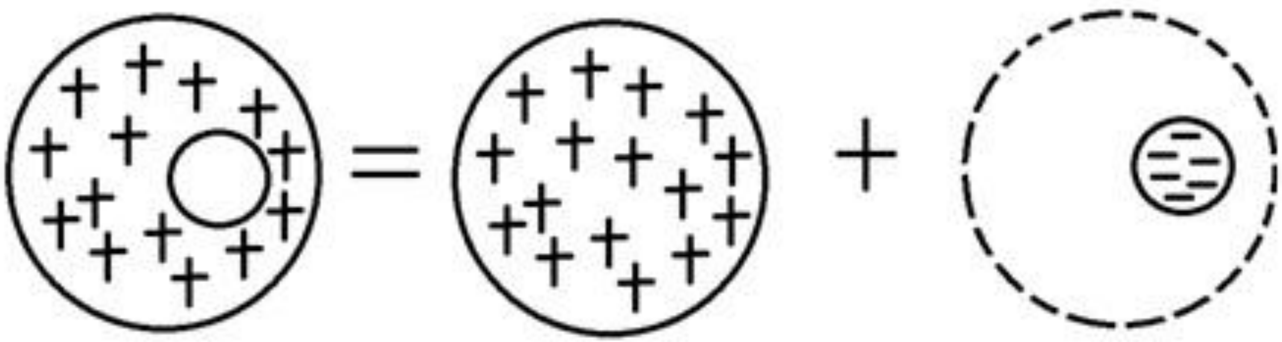


Electrostatics = Coulomb's law+ super position principle

- The electrical interaction between two charges is independent of the presence of third charge.. Remarkable property

Superposition principle……

- Charged sphere with cavity

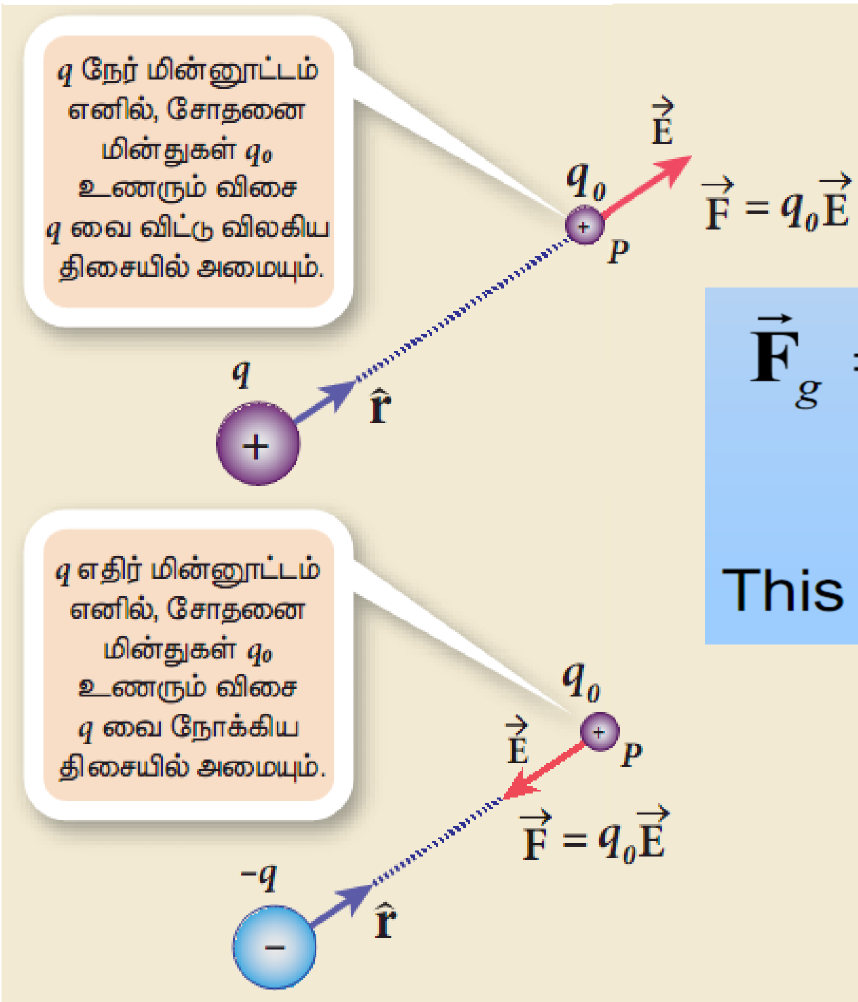


Interaction mechanism…

- Action at a distance……. (Newton)

- Keep one charge in earth and other charge in Pluto. There is an instantaneous interaction (classically..)
- Relativistically (it takes time $t=d/c$)
- Faraday – Electric field

- Electric field – At every point in space, unique magnitude and unique direction



$$\vec{\mathbf{F}}_g = m\vec{\mathbf{g}} \qquad \vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$$

This is easiest way to picture field

- Electric field – Force per unit charge

- Electric field is function of source charge and distance between source and field point.

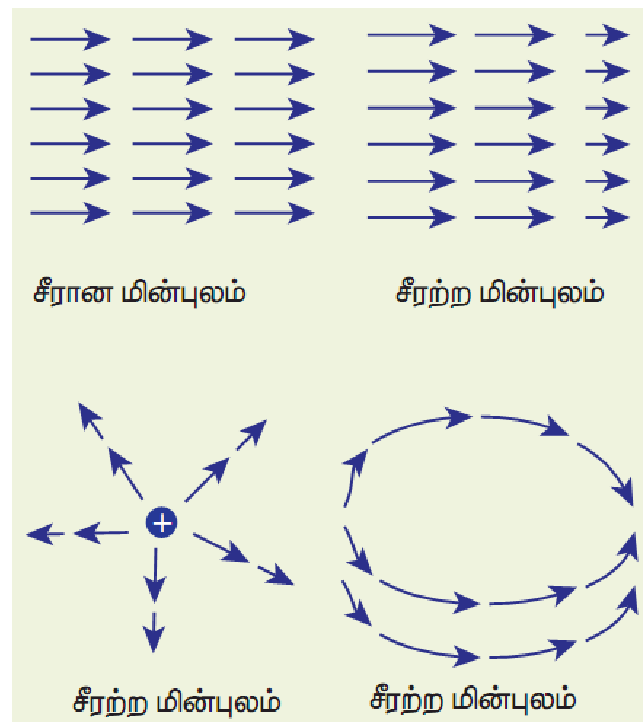
- Independent of test charge..

- Obeys superposition principle

Uniform and non uniform electric field

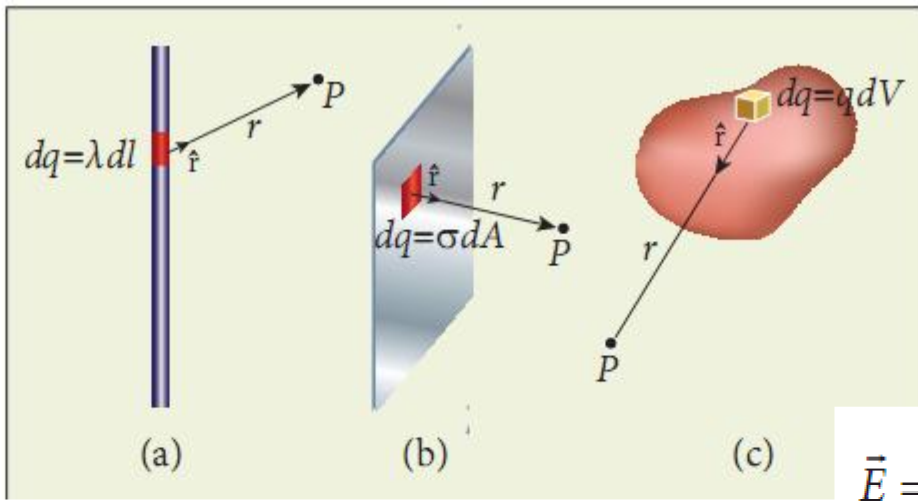
- Uniform electric field – Constant vector at every point in space

- Non-uniform– Vary in direction, magnitude and time



Electric field due to continuous charge distribution

- Summation – integration
- Point charge – infinitesimal charge element



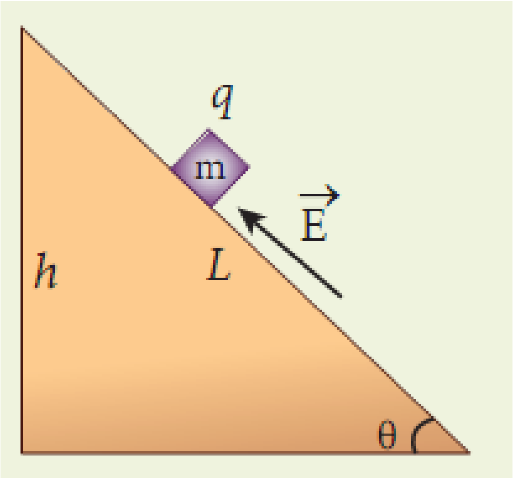
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda dl}{r^2}\hat{r}$$

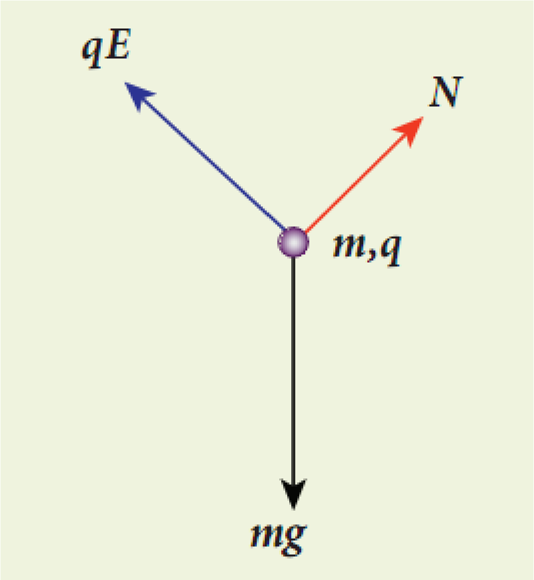
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\hat{r} \qquad (1.10)$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_\circ}\int \frac{\sigma da}{r^2}\hat{r}$$

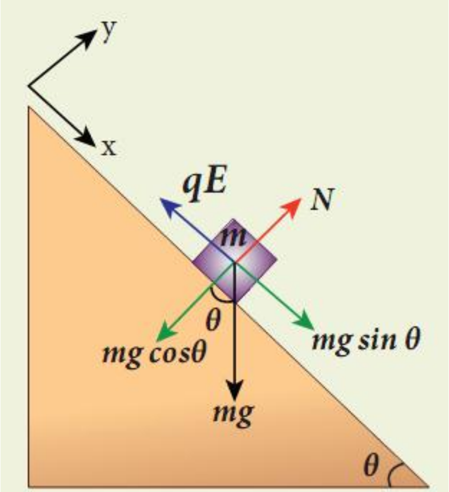
$$\vec{E} = \frac{1}{4\pi\varepsilon_\circ}\int \frac{\rho dV}{r^2}\hat{r}$$

Mechanics + electricity= marriage





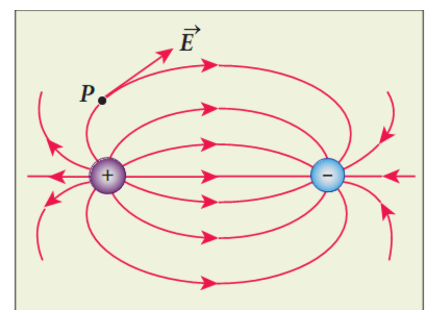
Free body diagram



Forces acting on the mass *m*

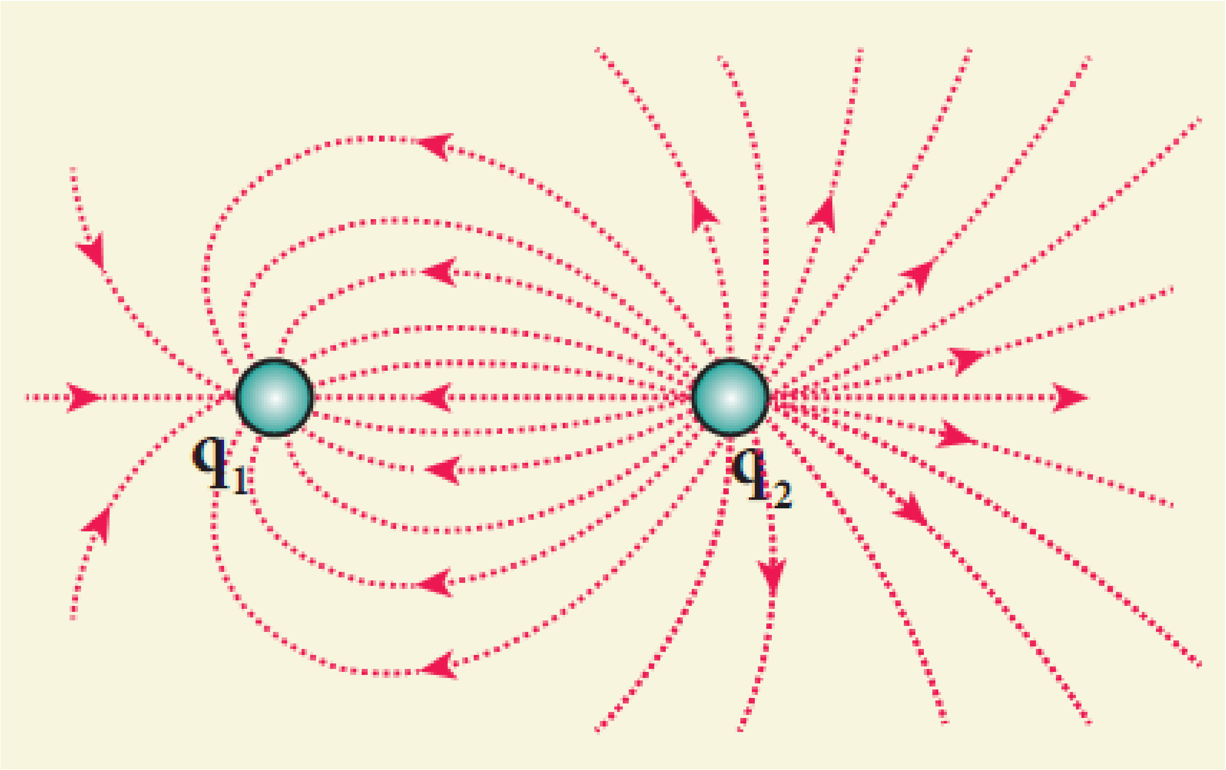
Electric field lines...

- Visual representation of electric field in some region of space
- Starts at positive charge, ends at negative or infinity
- Electric field at a point is tangent to the electric field line at that point
- Never intersect each other
- Denser field lines – Stronger E
- $N \propto q$



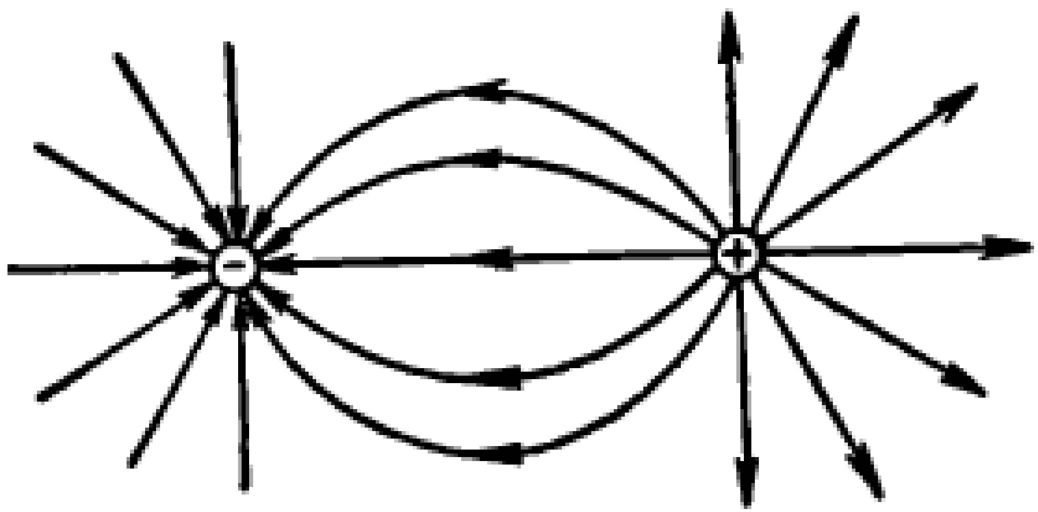
Ratio of $\left|\frac{q_1}{q_2}\right| = \frac{N_1}{N_2}$

- N_2= No. of lines starts from q_1
- N_1 = No. of lines connects q_1 and q_2



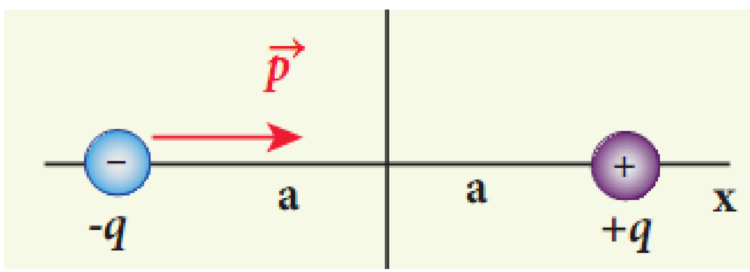
Electric field lines of equal and opposite charge

- What is wrong In this picture?



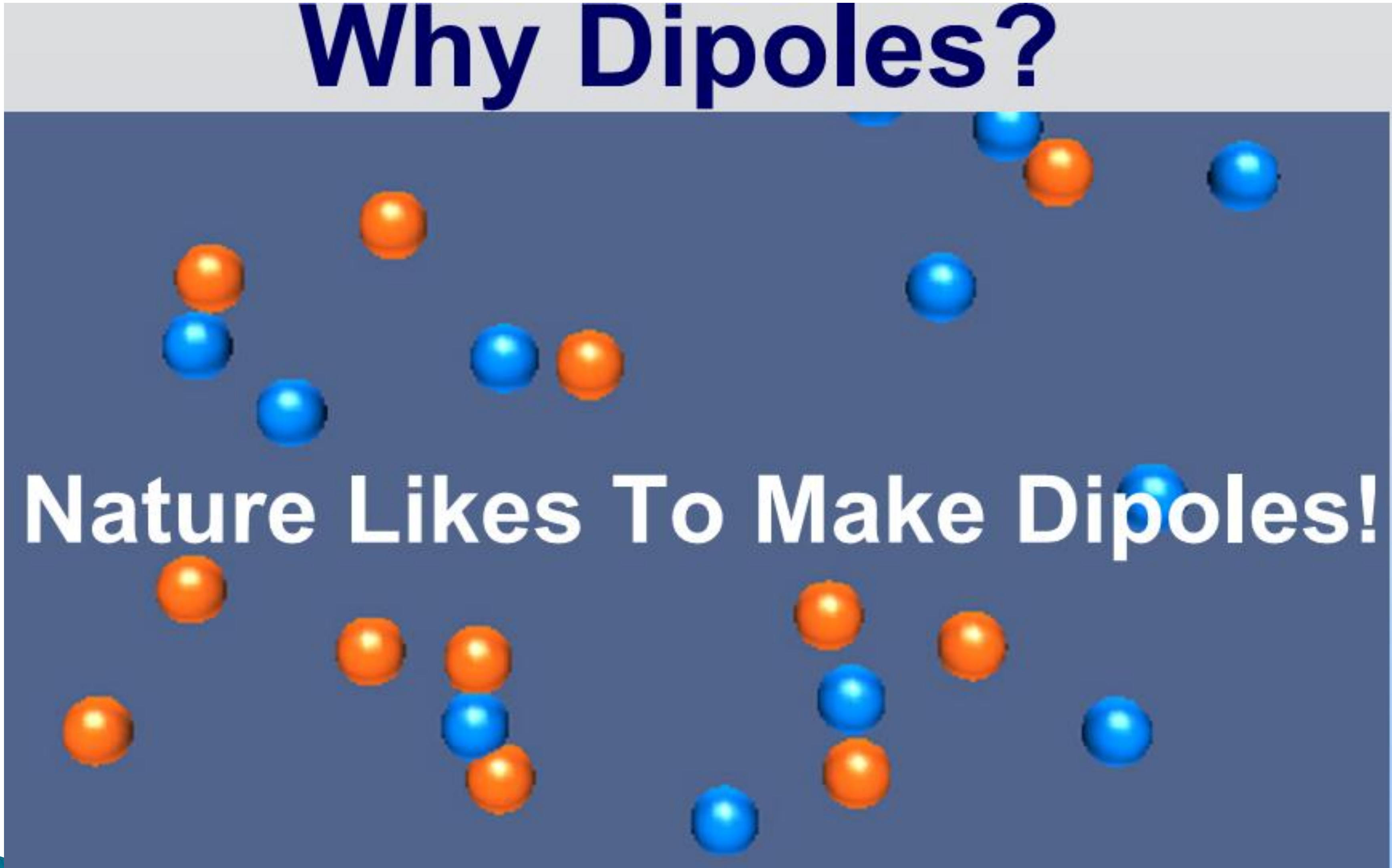
Electric dipole

- Two equal and opposite charges separated by a distance constitutes electric dipole….
- For two equal and opposite charges, Why dipole moment vector points from –q to +q ?

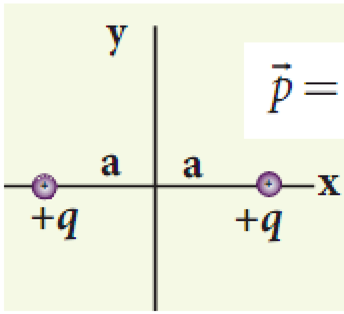


$$\vec{p} = \sum_{i=1}^{n} q_i \vec{r}_i$$

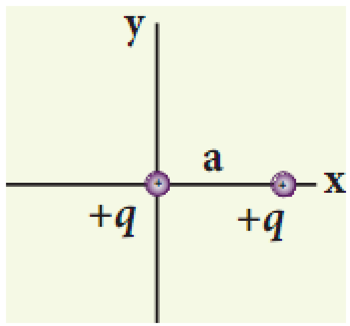
- Electric dipole moment is defined for one, two, three or any number of charges…


Why Dipoles?
Nature Likes To Make Dipoles!

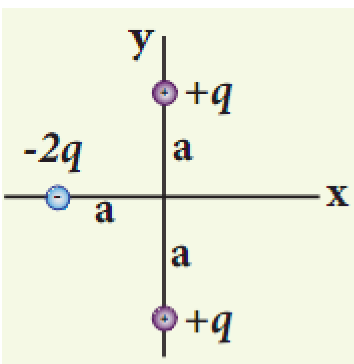
Few examples...



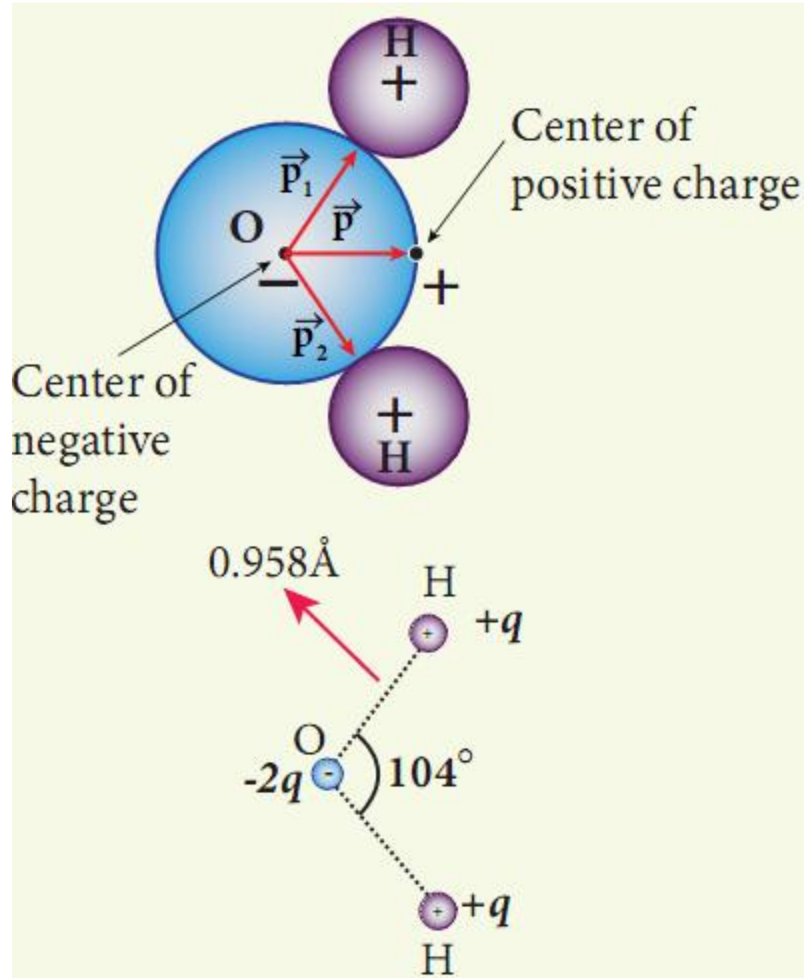
$$\vec{p} = (+q)(a\hat{i}) + (+q)(-a\hat{i}) = 0$$



$$\vec{p} = qa\hat{i}$$



$$\vec{p} = -2qa(-\hat{i}) + qa\hat{j} + qa(-\hat{j})$$
$$= 2qa\hat{i}$$

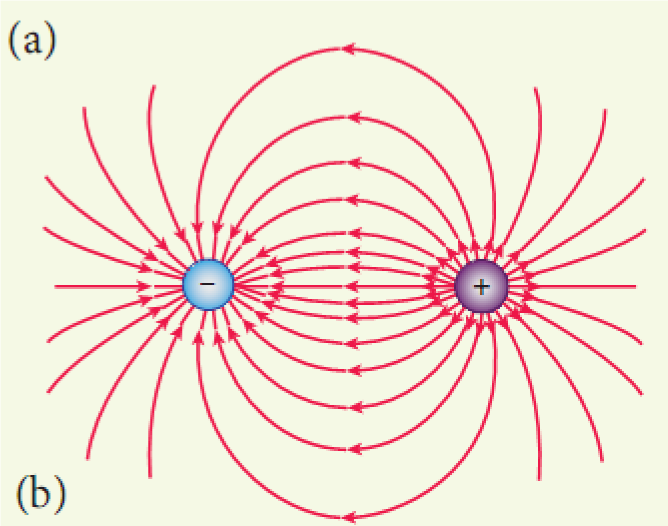


Electric field due to dipole

- Field is weaker at Equatorial than axial

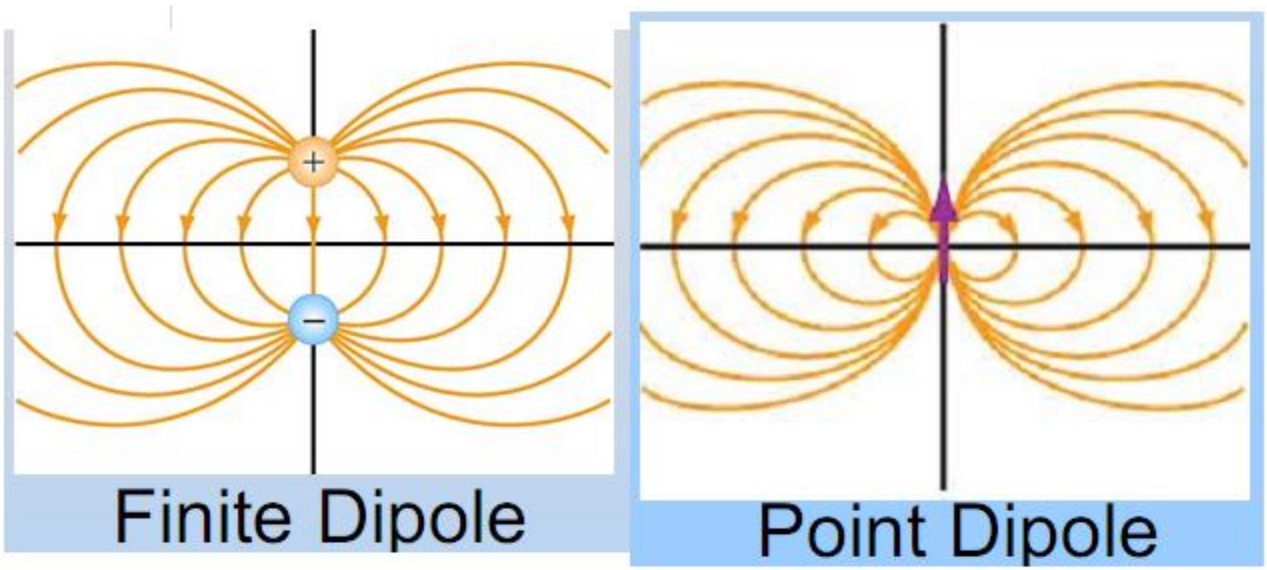
$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0} \frac{2\vec{p}}{r^3} \qquad (r >> a) \quad (1.17)$$

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_\circ} \frac{\vec{p}}{r^3} \qquad (r >> a) \qquad (1.21)$$



- Electric field lines are less denser at equatorial plane
- For point dipole $(2a \to 0, q \to \infty\ and\ p = finite)$ the expression is exact.

Finite dipole Vs Point dipole



Electric field lines for (a) a finite dipole and (b) a point dipole.

- For monopole(point charge) $E \propto \frac{1}{r^2}$
- For dipole $E \propto \frac{1}{r^3}$
- For quadrapole $E \propto \frac{1}{r^4}$

and so on……

Dipole in electric field

$$\vec{\tau} = \overrightarrow{OA} \times \left(-q\vec{E}\right) + \overrightarrow{OB} \times q\vec{E} \qquad (1.22)$$

- In uniform electric field – Net torque, no net force
- In non uniform electric field – Net torque and net force

- Think?? A dipole is placed in the following field.. In which case it experiences net torque and net force?
- (a)point charge (b)charged infinite plane
- (b)charged infinite wire (d) charged hollow sphere (both inside and outside)

Electrostatic force is conservative force....

- So we could define potential energy..

- Gravitational force is conservative force – Gravitational potential energy
- Spring force is conservative force – Elastic potential energy $U = \frac{1}{2}kx^2$
- Frictional force is not conservative.

Electrostatic potential

- Work done to bring a unit positive charge at constant velocity from infinity to the point **P**

$$V_P = -\int_{\infty}^{P} \vec{E} \cdot d\vec{r}$$

- Potential difference

$$V_P - V_R = \Delta V = \int_{R}^{P} -\vec{E} \cdot d\vec{r}$$

- potential = potential energy per unit charge

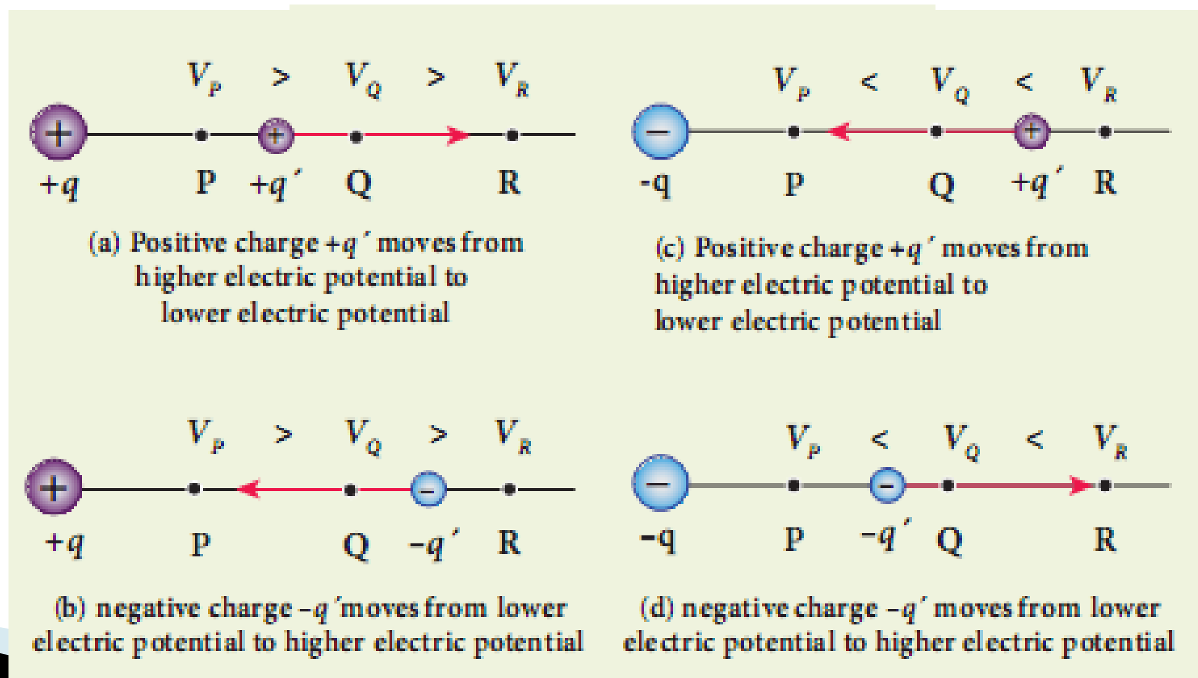
- Explaining the motion of the charges using potential(scalar approach) is easier than electric field(vector approach)..

- Potential due to point charge

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad (1.33)$$

Motion of the charges..

- Positive charge always moves from higher electric potential to lower electric potential
- Negative charge always moves from lower electric potential to higher electric potential

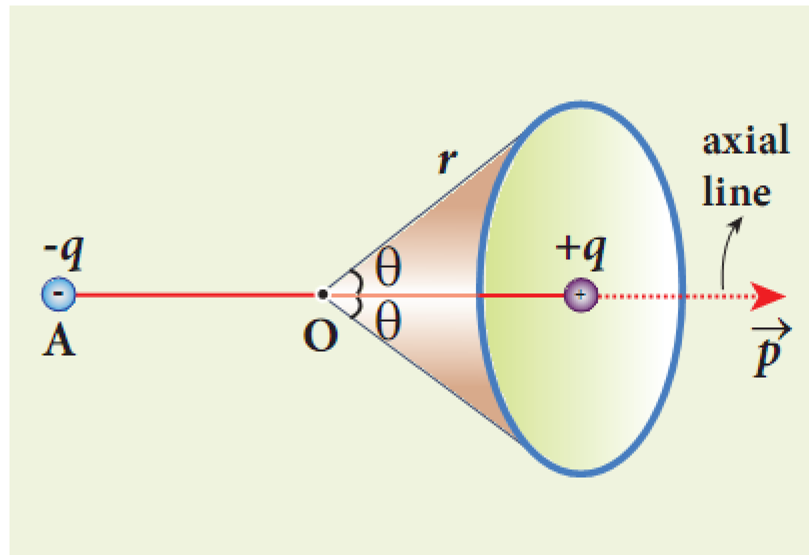


Electric potential

- For point charge – V is spherically symmetric
- For dipole – V is axially symmetric

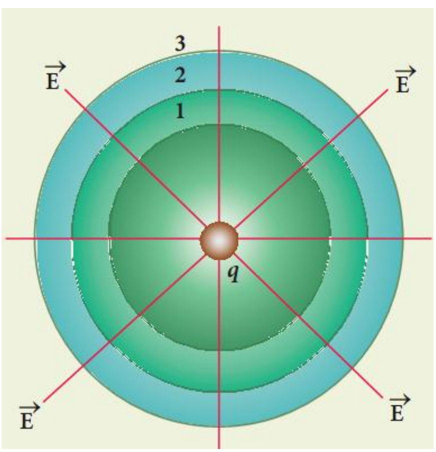
$$V = \frac{1}{4\pi\varepsilon_\circ}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

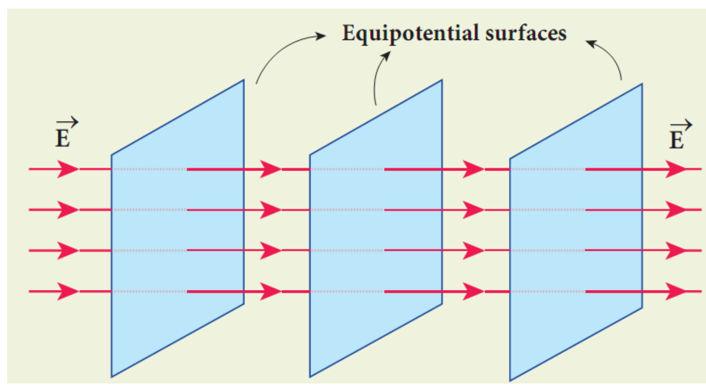
- Blue circle has same potential. Because it subtends same angle θ to the point O.



Equipotential surface

- V is constant over equipotential surface
- Electric field is normal to equipotential surface. No tangential electric field.



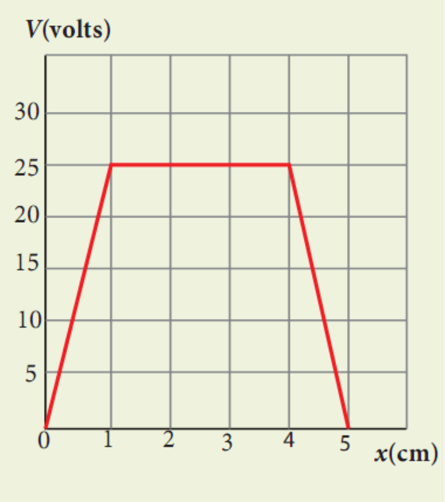


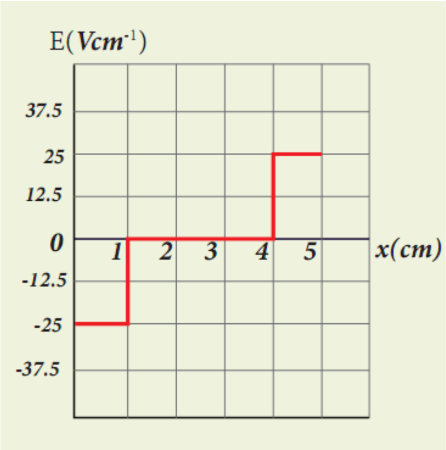
Relation between $\vec{E}$ and V

- In one dimension

$$E = -\frac{dV}{dx}$$

- Example 1.14





In 3-dimension…

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

- Derive Electric field of point charge from potential..
- $V = \frac{q}{r} = \frac{q}{\sqrt{x^2+y^2+z^2}}$
- $\frac{\partial V}{\partial x} = -\frac{qx}{(x^2+y^2+z^2)^{\frac{3}{2}}}; \frac{\partial V}{\partial y} = -\frac{qy}{(x^2+y^2+z^2)^{\frac{3}{2}}}\ \frac{\partial V}{\partial z} = -\frac{qz}{(x^2+y^2+z^2)^{\frac{3}{2}}}$
- $\vec{E} = -(-\frac{qx}{(x^2+y^2+z^2)^{\frac{3}{2}}}\hat{\imath} - \frac{qy}{(x^2+y^2+z^2)^{\frac{3}{2}}}\hat{\jmath} - \frac{qz}{(x^2+y^2+z^2)^{\frac{3}{2}}}\hat{k})$
- $\vec{E} = q\frac{x\hat{\imath}+y\hat{\imath}+z\hat{\imath}}{(x^2+y^2+z^2)^{\frac{3}{2}}}=q\frac{\vec{r}}{r^3} = \frac{q}{r^2}\hat{r}$

Potential energy…

- For two point charges $U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$
- For three point charges $$U = \frac{1}{4\pi\varepsilon_\circ}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right) \quad (1.48)$$
- It is equal to total external work required to assemble these configuration..
- It is independent of the manner in which the configuration is achieved.

Electrostatic potential energy of a dipole in a uniform electric field

$$W = \int_{\theta'}^{\theta} \tau_{ext} d\theta \qquad (1.49)$$

$$W = \int_{\theta'}^{\theta} pE \sin\theta \, d\theta$$

$$W = pE\left(\cos\theta' - \cos\theta\right)$$

$$U = -pE\cos\theta = -\vec{p} \cdot \vec{E}$$

Conservation of energy

- Electrostatic potential energy + Kinetic energy = constant

- It is similar like gravitational case

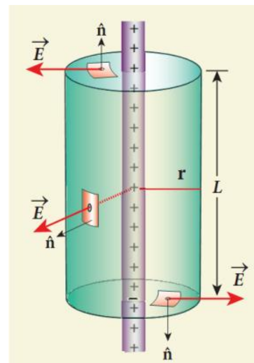
Gauss law

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_\circ}$$

- Total electric flux in a closed surface depends only on the net charges enclosed by this surface.
- It is a Consequence of inverse square law
- Gaussian surface : calculation tool.
- Shape of Gaussian surface depends on symmetry of the charge configuration..

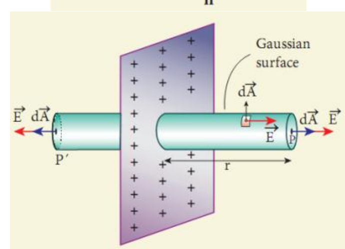
Cleverness in Gauss law

- How to take out the electric field $\vec{E}$ from the flux integration.
- Infinite wire:



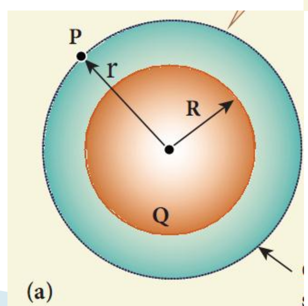
$$E \int_{\text{Curved surface}} dA = \frac{\lambda L}{\varepsilon_\circ}$$

- Infinite plane:



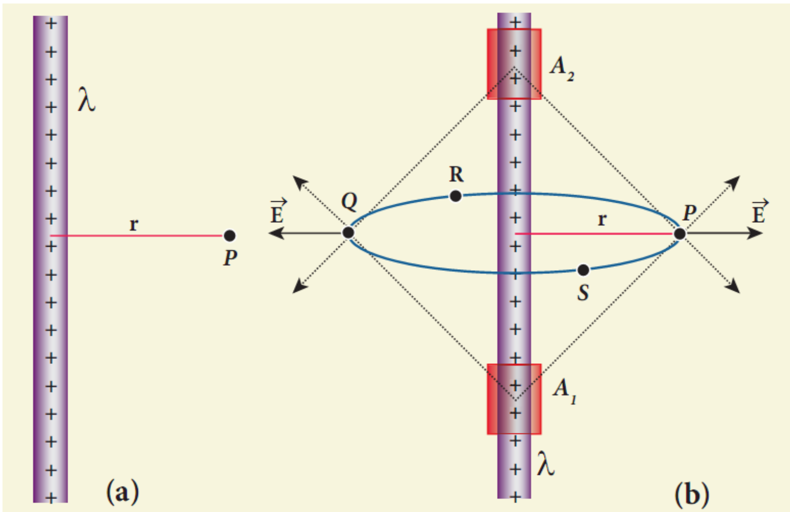
$$2E \int_{P} dA = \frac{\sigma A}{\varepsilon_0}$$

- Point charge:



$$E \oint_{\text{Gaussian surface}} dA = \frac{Q}{\varepsilon_\circ}$$

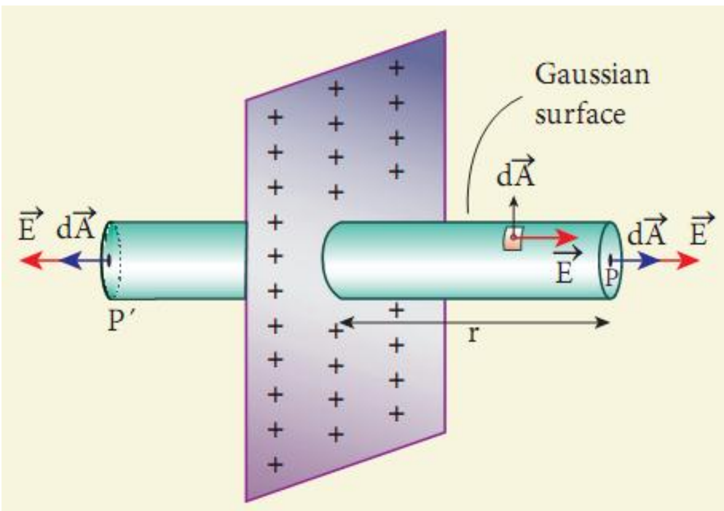
Infinitely long charged wire



$$\vec{E} = \frac{1}{2\pi\varepsilon_\circ}\frac{\lambda}{r}\hat{r}$$

The equation (1.67) is true only for an infinitely long charged wire. For a charged wire of finite length, the electric field need not be radial at all points. However, equation (1.67) for such a wire is taken approximately true around the mid-point of the wire and far away from the both ends of the wire

Charged Infinite plane



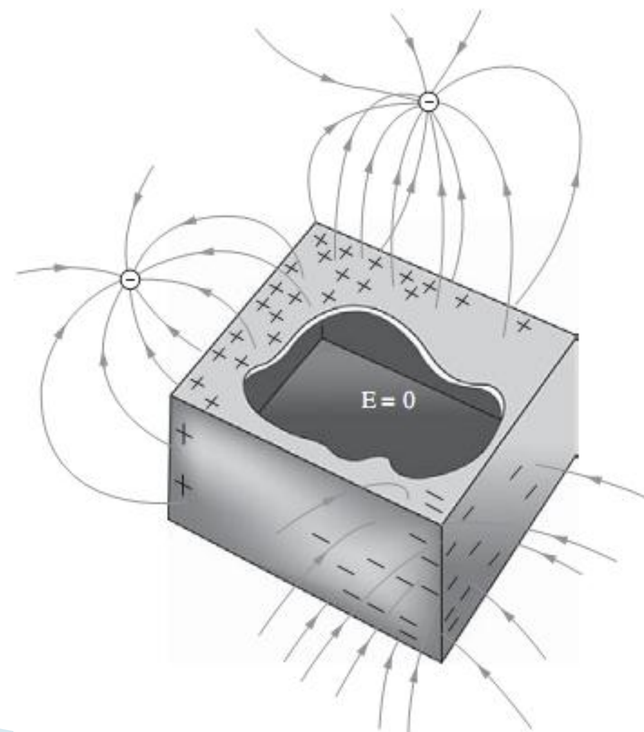
$$\vec{E} = \frac{\sigma}{2\varepsilon_\circ}\hat{n}$$

For a finite charged plane sheet, equation (1.71) is approximately true only in the middle region of the plane and at points far away from both ends.

Hollow sphere of charge Q

It is equivalent to point charge of Q located at the center of the sphere.

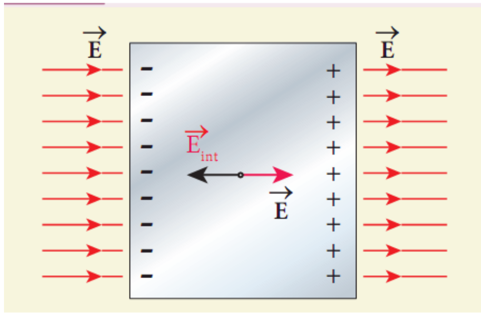
Inside the hollow sphere, $\vec{E}$ is zero. It is property of inverse square law. Faraday Cage uses this principle.



Symmetry	**System**	**Gaussian Surface**
Cylindrical	Infinite rod	Coaxial Cylinder
Planar	Infinite plane	Gaussian "Pillbox"
Spherical	Sphere, Spherical shell	Concentric Sphere

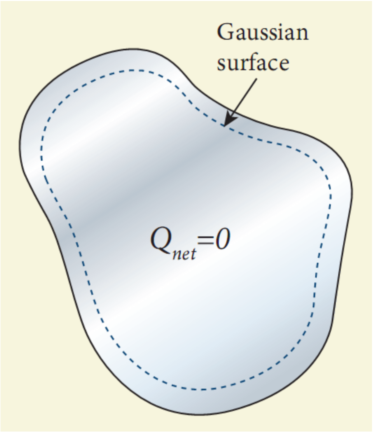
Electrostatics of Conductors…

- (1) $\vec{E}$ is zero everywhere inside the conductor(experimental fact..)



- (2) There is no net charge inside the conductors(Gauss law).

(3) outside $E = \frac{\sigma}{\varepsilon_\circ}$ and perpendicular to the surface of conductor (Gauss law)



(4) surface of the conductor is always at equipotential

Dielectrics or insulators

- Polar molecules – Permanent dipole moment. But net dipole moment is zero due to thermal agitation. In the presence of external $\vec{E}$, net dipole moment
  - Ex: H_2O, N_2O, HCl, NH_3.

- Non polar molecule – only induced dipole moment when there is external electric field

  Ex: H_2, O_2, CO_2

Polarisation

- Dipole moment per unit volume of the dielectric.
- For linear isotropic material

$$\vec{P} = \chi_e \vec{E}_{ext}$$

- Linear : $P \propto E$. Note that First power of E
- Isotropic: direction of $\vec{P}$ is in the direction of $\vec{E}$ and magnitude of $\vec{P}$ independent of direction of $\vec{E}$
- χ_e – Electric Susceptibility.Scalar (isotropic) and tensor (anisotropic)

Capacitors

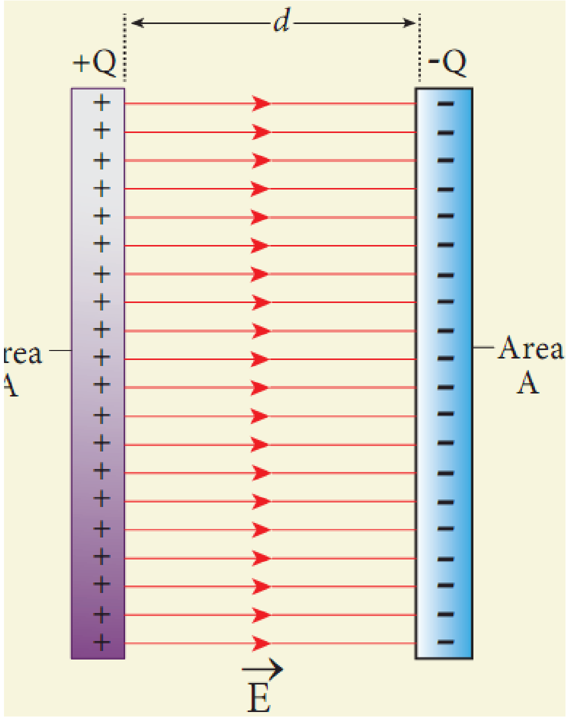
$$C = \frac{Q}{V}$$

$$Q = CV$$

- It is a devise to store electric charge and electrical energy..
- Storage of charge implies total charge is in one plate of the capacitor.

- For parallel plate

$$C = \frac{Q}{V} = \frac{Q}{\left(\frac{Qd}{A\varepsilon_0}\right)} = \frac{\varepsilon_0 A}{d}$$



Energy stored in the capacitor

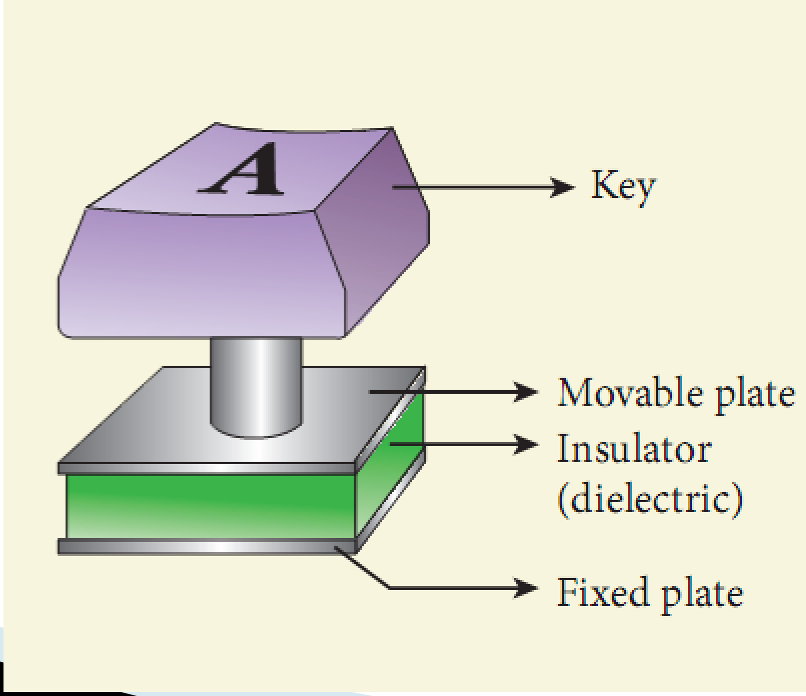
- In parallel plate capacitor $$U_E = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$
- Energy density $$u_E = \frac{1}{2}\varepsilon_\circ E^2$$

Few uses of capacitors

Effects of dielectric in the parallel plate capacitor

Table 1.2

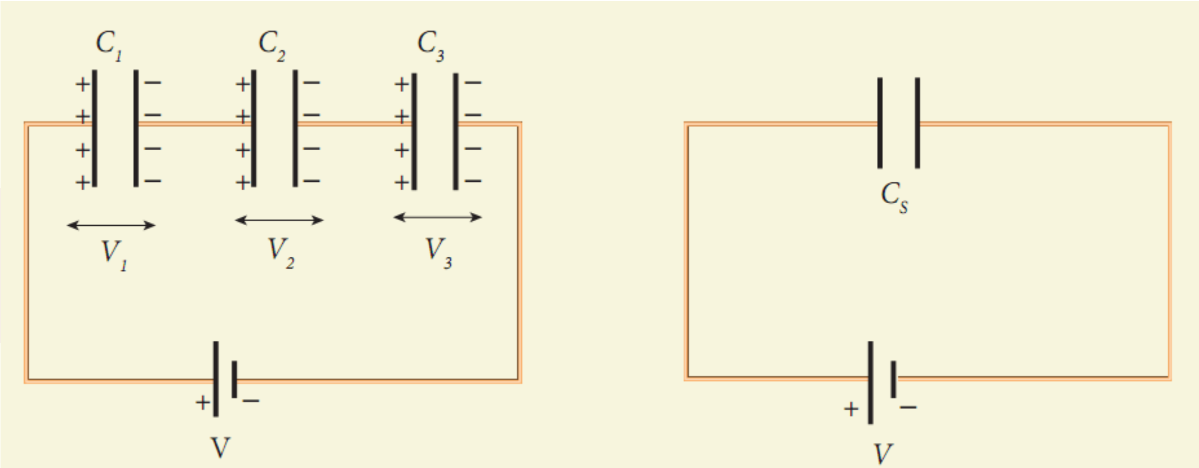
S. No	Dielectric is inserted	Charge Q	Voltage V	Electric field E	Capacitance C	Energy U
1	When the battery is disconnected	Constant	decreases	Decreases	Increases	Decreases
2	When the battery is connected	Increases	Constant	Constant	Increases	Increases



Capacitors series and parallel

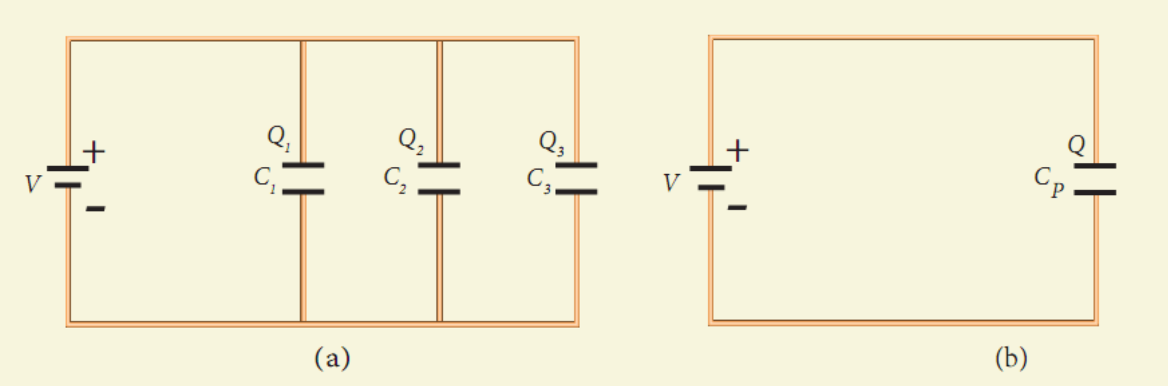
- For series:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



- For parallel:

$$C_P = C_1 + C_2 + C_3$$



Distribution of charges in the conductor

- Less curvature – more charge density
- More curvature– less charge density

$$\sigma r = \text{constant}$$

- Action at a point –Corona discharge – Lightning arrestor works in this principle

Vande Graaff generator

- Can be produced voltage upto $10^7 V$.