Magnetism

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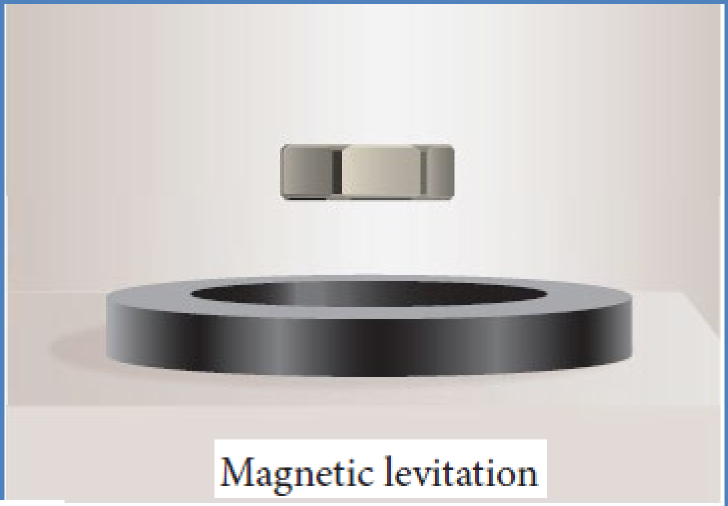
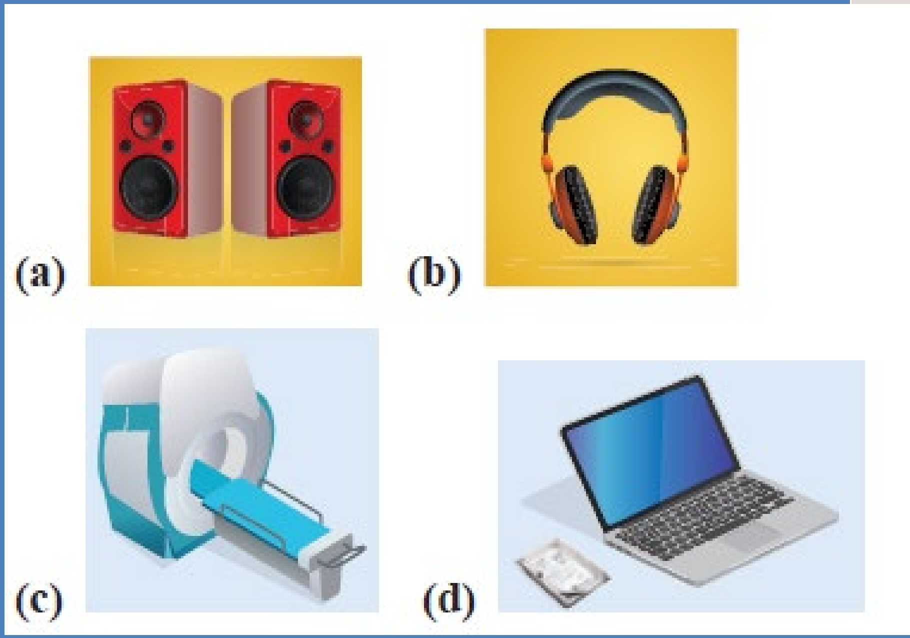
Magnets

- Magnets!





Earth's Magnetic field

- Earth behaves like huge (gigantic) bar magnet
- Freely suspended bar magnet or a magnetic compass comes to rest in a position which is approximately along the geographical north-south direction of the Earth. This means the magnetic pole of the Earth and geographic pole of the Earth do not coincide.
- If the north pole of the magnet compass points to the north then the magnetic pole at the northern end of the Earth should be a south pole
- This implies, the southern end of the Earth should be north pole.

Definition

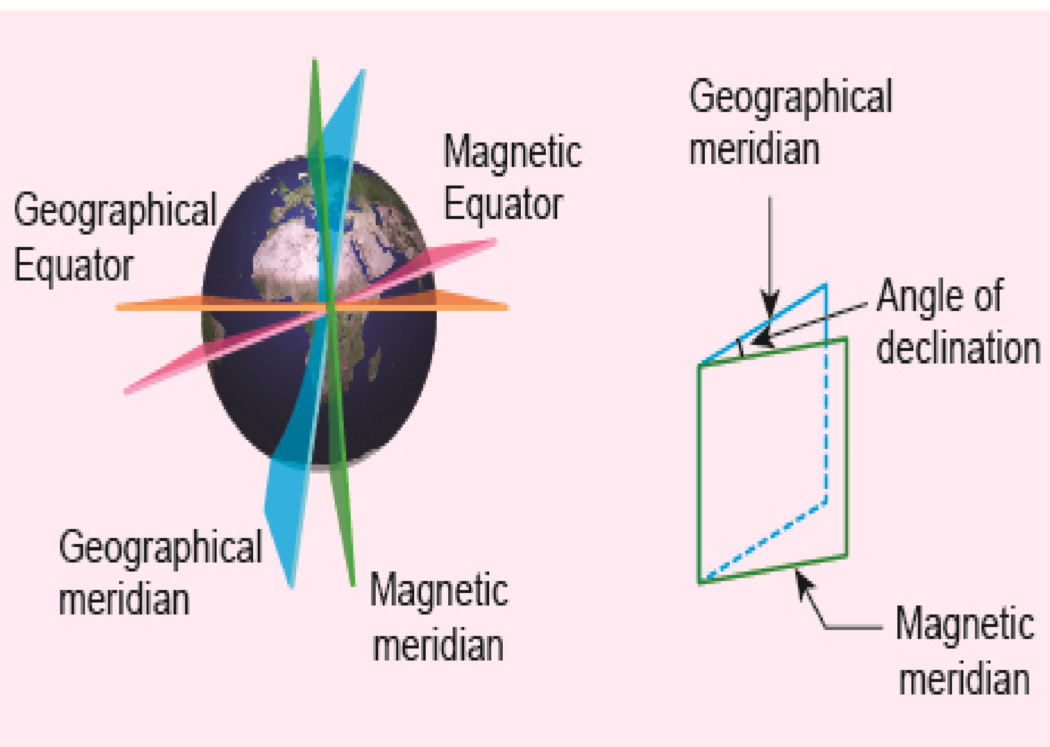
- Geographic meridian at a point (place) is defined as a plane containing that point and the geographic north and geographic south pole of the Earth.
- Magnetic meridian at a point (place) is defined as a plane containing that point (place) and the magnetic north and magnetic south pole.
- The quantities that are completely determine Earth's magnetic field are defined as the elements of the Earth's magnetic field

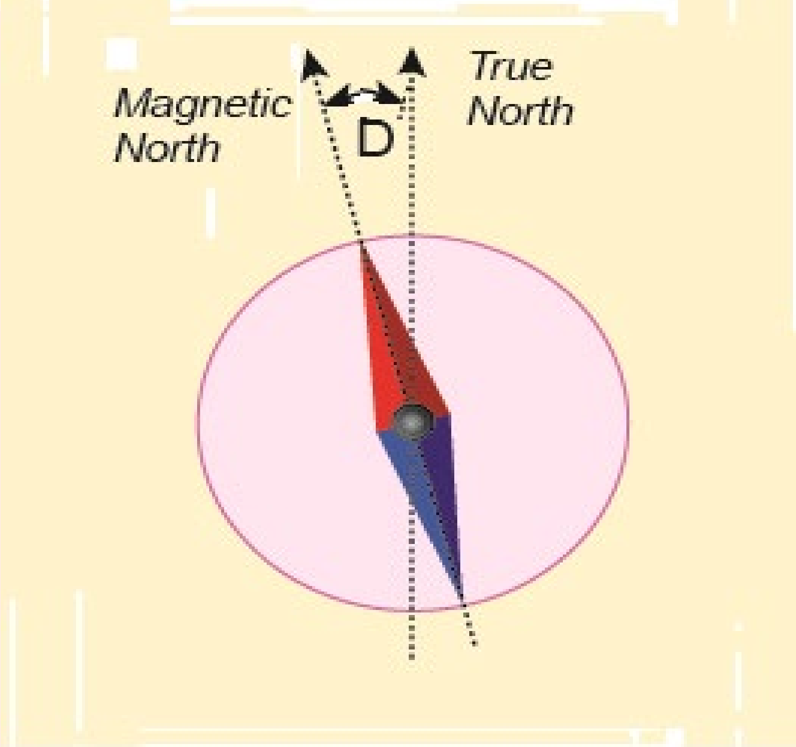
Geomagnetism

- To completely specify the magnetic field at any point on the surface of the Earth, we have to specify three elements at a point
  - Declination or magnetic declination (D)
  - Dip or magnetic inclination (I)
  - Horizontal component of Earth's magnetic field (B_H)

Angle of declination

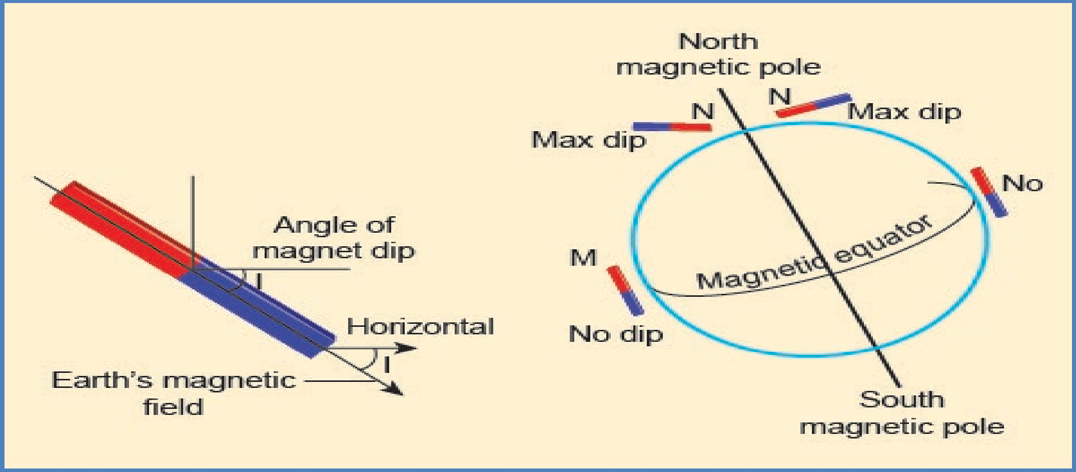
- The angle between geographical meridian and geographical meridian at a point (place) is called declination ata point (place)

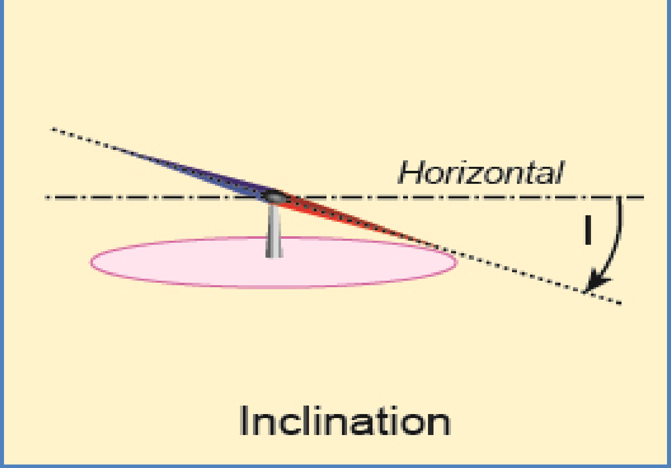




Dip or inclination

- The angle between a freely suspended magnet compass and the horizontal plane at a point (place) is defined as the dip or inclination at that point (place)
- In the Northern hemisphere, north pole of the magnetic compass dip downward and in Southern hemisphere, south pole of the magnetic compass dip downward





Horizontal component of Earth's magnetic field

- The component of Earth's magnetic field along the horizontal direction in the magnetic meridian.

Let B_E be the net Earth's magnetic field at a point P on the surface of the Earth. B_E can be resolved into two perpendicular components.

Horizontal component $B_H = B_E \cos I$

Vertical component $B_V = B_E \sin I$

$$\tan I = \frac{B_V}{B_H}$$

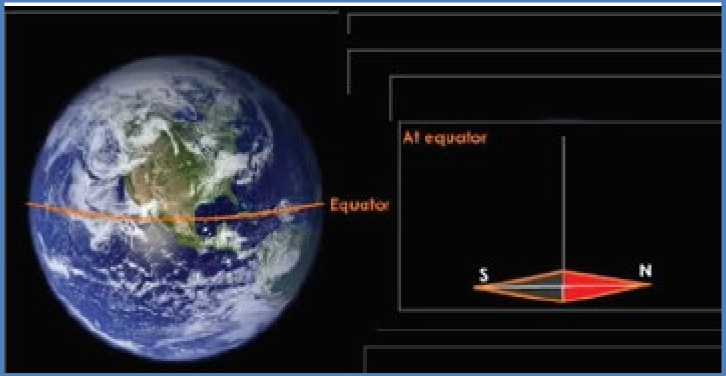
The resultant magnetic field of the Earth is

$$B_E = \sqrt{B_H^2 + B_V^2}$$

- **At magnetic equator** – Earth's magnetic field is parallel to the surface of the Earth (i.e., horizontal) which implies that the needle of magnetic compass rests horizontally at an angle of dip $I = 0^{o}$

$$B_H = B_E$$
$$B_V = 0$$

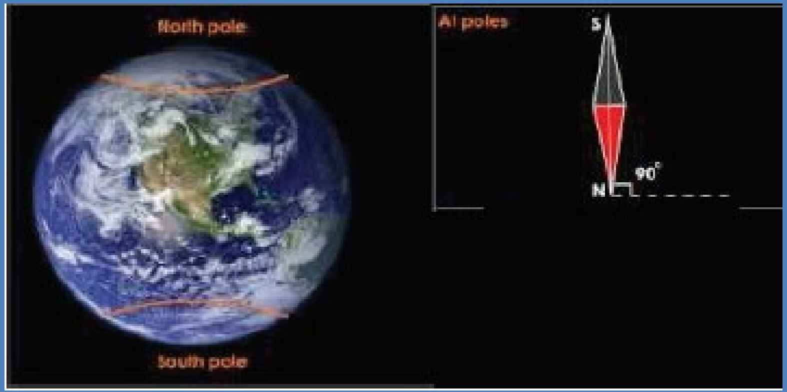
This implies that the horizontal component is maximum at equator and vertical component is zero at equator.



- A magnetic poles – Earth's magnetic field is perpendicular to the surface of the Earth (i.e., vertical) which implies that the needle of magnetic compass rests vertically at an angle of dip $I = 90^{o}$

$$B_H = 0$$
$$B_V = B_E$$

This implies that the vertical component is maximum at poles and horizontal component is zero at poles.



Magnetic dipole moment

- **The magnetic dipole moment is defined as the product of its pole strength and magnetic length**. It is a vector quantity.

$$\vec{p}_m = q_m \vec{d}$$

where $\vec{d}$ is the vector drawn from south pole to north pole and its magnitude $|\vec{d}| = 2l$.

The SI unit of magnetic moment is A m^2. Note that the direction of magnetic moment is from South pole to North pole.

Note

(i) Pole strength is a scalar quantity with dimension $[M^0 L T^0 A]$. Its SI unit is $N\,T^{-1}$ (newton per tesla) or A m (ampere-metre).

(ii) Like positive and negative charges in electrostatics, north pole of a magnet experiences a force in the direction of magnetic field while south pole of a magnet experiences force opposite to the magnetic field.

(iii) Pole strength depends on the nature of materials of the magnet, area of cross-section and the state of magnetization.

(iv) If a magnet is cut into two equal halves along the length then pole strength is reduced to half.

(v) If a magnet is cut into two equal halves perpendicular to the length, then pole strength remains same.

(vi) If a magnet is cut into two pieces, we will not get separate north and south poles. Instead, we get two magnets. In other words, isolated monopole does not exist in nature.

Magnetic field

- Magnetic field is the region or space around every magnet within which its influence can be felt by keeping another magnet in that region.
- The magnetic field at a point is defined as a force experienced by the bar magnet of unit pole strength.

$$\vec{B} = \frac{1}{q_m} \vec{F}$$

Its unit is N A^{-1} m^{-1}

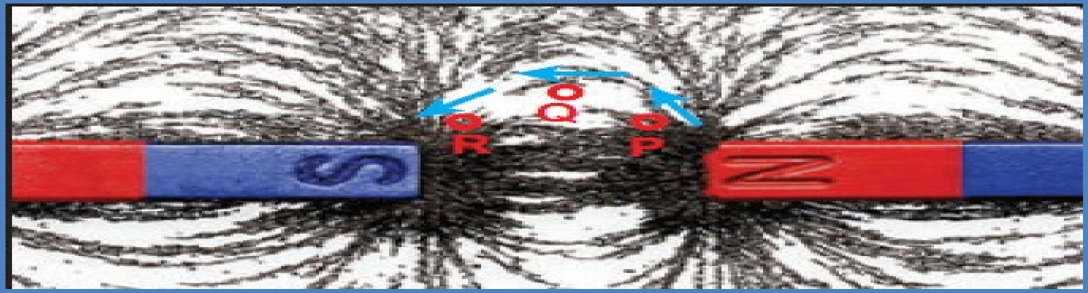
Properties of a bar magnet

- 1. A freely suspended bar magnet will always point along the north-south direction.
- 2. A magnet attracts another magnet or magnetic substances towards itself. The attractive force is maximum near the end of the bar magnet. When a bar magnet is dipped into iron filling, they cling to the ends of the magnet.
- 3. When a magnet is broken into pieces, each piece behaves like a magnet with poles at its ends.
- 4. The pole strength of two poles of a magnet is equal of strengths.
- 5. The length of the bar magnet is called geometrical length and the length between two magnetic poles in a bar magnet is called magnetic length. Magnetic length is always slightly smaller than geometrical length.

$$\frac{\text{Magnetic length}}{\text{Geometrical length}} = \frac{5}{6} = 0.833$$

Properties of Magnetic field lines

- 1. Magnetic field lines are continuous closed curves. The direction of magnetic field lines is from North pole to South pole outside the magnet and South pole to North pole inside the magnet.
- 2. The direction of magnetic field at any point on the curve is known by drawing tangent to the magnetic line of force at that point. In the Figure, the tangent drawn at points P,Q and R gives the direction of magnetic field at that point.
- 3. Magnetic field lines never intersect each other. Otherwise, the magnetic compass needle would point towards two directions, which is not possible.



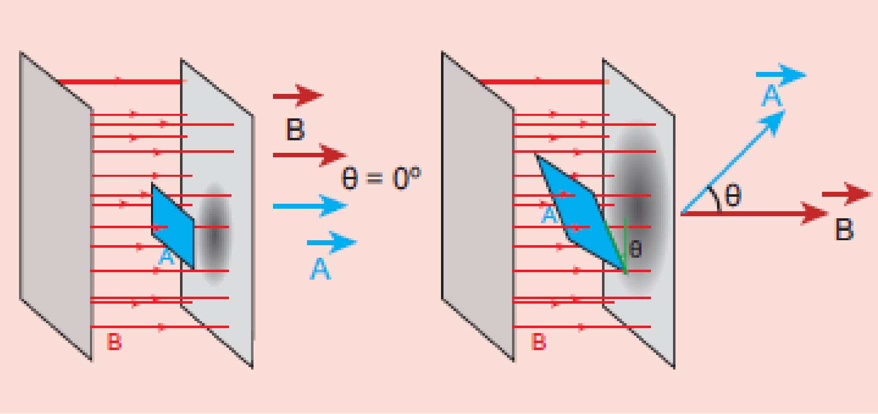
- 4. The degree of closeness of the field lines determines the relative strength of the magnetic field. The magnetic field is strong where magnetic field lines crowd and weak where magnetic field lines thin out.

Magnetic flux

The number of magnetic field lines crossing per unit area is called magnetic flux Φ_B. Mathematically, the magnetic flux through a surface of area A in a uniform magnetic field is defined as

$$\Phi_B = \vec{B}.\vec{A} = BA\cos\theta = B_{\perp} A$$

where θ is the angle between $\vec{B}$ and $\vec{A}$



Special cases

(a) When $\vec{B}$ is normal to the surface i.e., $\theta = 0^{\circ}$, the magnetic flux is $\Phi_B = BA$ (maximum).

(b) When $\vec{B}$ is parallel to the surface i.e., $\theta = 90^{\circ}$, the magnetic flux is $\Phi_B = 0$.

Magnetic flux (cont…) and Flux density

- If the magnetic field is not uniform, then

$$\Phi_B = \int \vec{B}.d\vec{A}$$

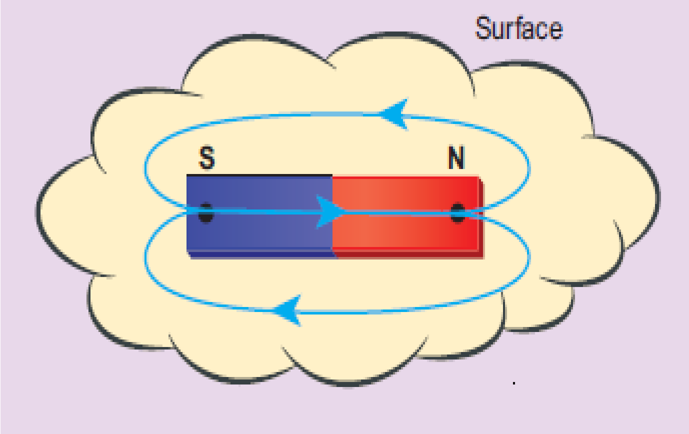
Magnetic flux is a scalar quantity. The SI unit for magnetic flux is weber, which is denoted by symbol Wb. Dimensional formula for magnetic flux is $\left[ML^2T^{-2}A^{-1}\right]$. The CGS unit of magnetic flux is Maxwell.

$$1 \text{ weber} = 10^8 \text{ maxwell}$$

The *magnetic flux density* can also be defined as the number of magnetic field lines crossing unit area kept normal to the direction of line of force. Its unit is Wb m^{-2} or tesla.

EXAMPLE 3.4

Calculate the magnetic flux coming out from the surface containing magnetic dipole (say, a bar magnet) as shown in figure.



Solution

Magnetic dipole is kept, the total flux emanating from the closed surface S is zero. So,

$$\Phi_B = \oint \vec{B}.d\vec{A} = 0$$

Here the integral is taken over closed surface. Since no isolated magnetic pole (called magnetic monopole) exists, this integral is always zero,

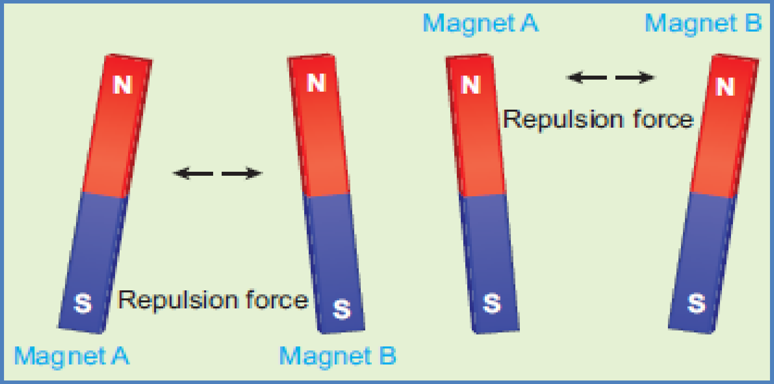
$$\oint \vec{B}.d\vec{A} = 0$$

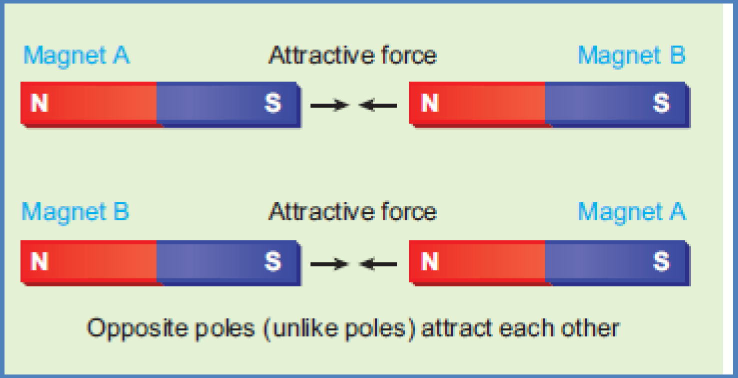
This is similar to Gauss's law in electrostatics.

Magnet – Monopole or dipole?

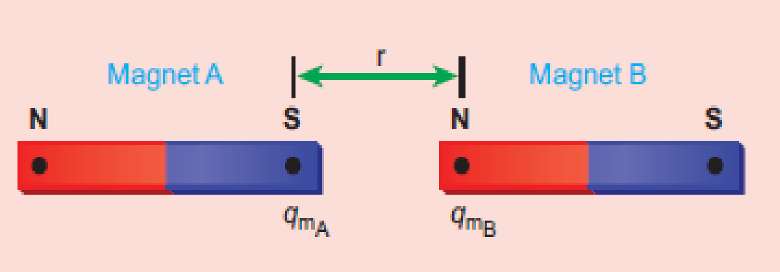
- For Magnets – no isolated monopole exists
- Only dipole exists
- In general, except isolated monopole, all the higher order moment exists for magnetism – dipole moment, quadrupole moment, octopole moment, etc. (often known as multipole expansion)

Coulomb's inverse square law in magnetism





Coulomb's law in magnetism - The force of attraction or repulsion between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them



$$\vec{F} = k\frac{q_{m_A} q_{m_B}}{r^2}\hat{r}$$

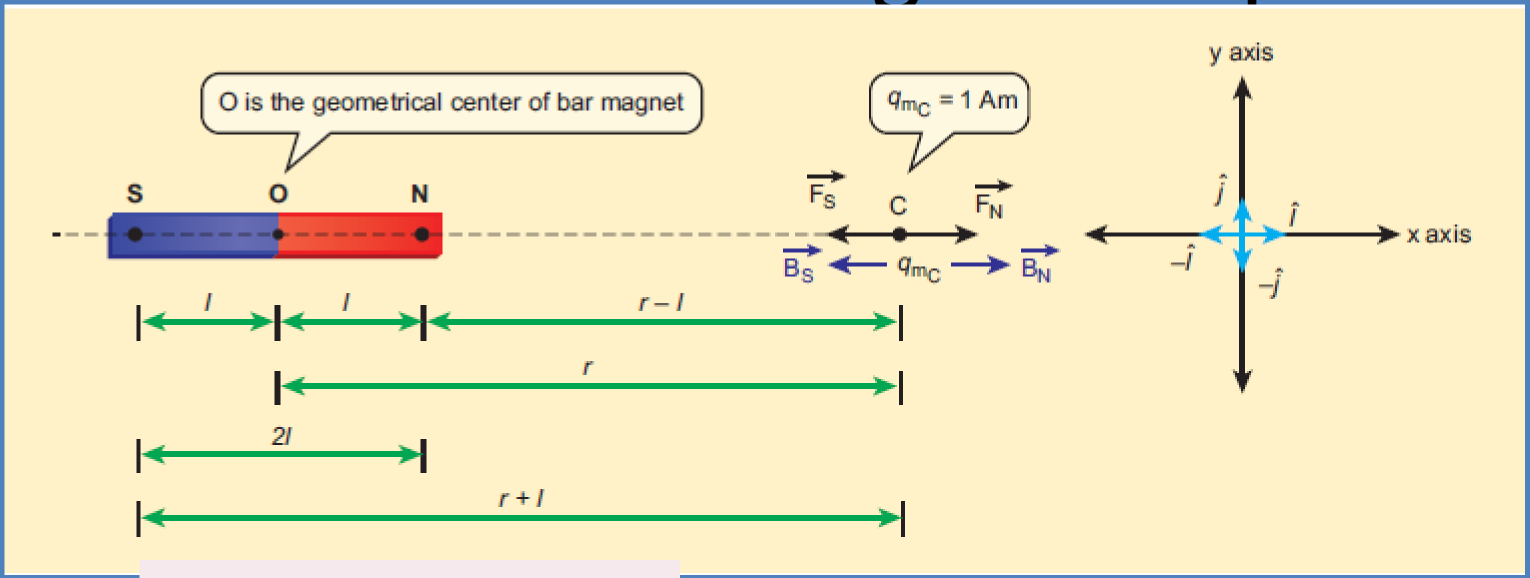
$$k = \frac{\mu_0}{4\pi} \approx 10^{-7}\, H m^{-1}$$

Dipole – magnetic field at a point

- Dipole - (a) Magnetic field at axial line

  (b) Magnetic field at equatorial line

Magnetic field at a point along the axial line due to magnetic dipole



$$\vec{B} = \frac{\mu_0 2r}{4\pi}\left(\frac{q_m \cdot (2l)}{\left(r^2 - l^2\right)^2}\right)\hat{i}$$

$$\vec{B}_{axial} = \frac{\mu_0}{4\pi}\left(\frac{2rp_m}{\left(r^2 - l^2\right)^2}\right)\hat{i}$$

$$\left|\vec{p}_m\right| = p_m = q_m \cdot 2l$$

$$\vec{B}_{axial} = \frac{\mu_0}{4\pi}\left(\frac{2p_m}{r^3}\right)\hat{i} = \frac{\mu_0}{4\pi}\frac{2}{r^3}\vec{p}_m$$

where $\vec{p}_m = p_m\hat{i}$.

Results from electrostatics

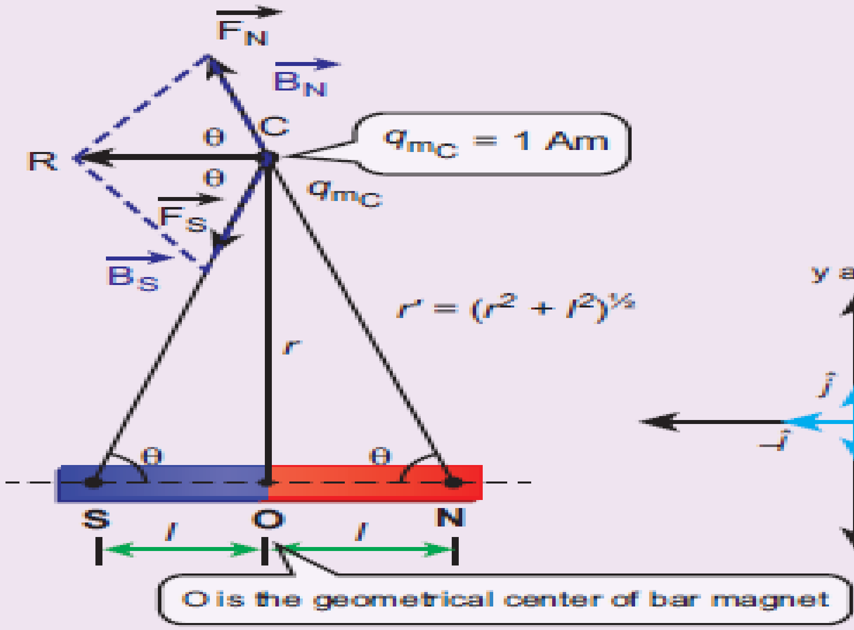
$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0} q\left(\frac{4ra}{\left(r^2 - a^2\right)^2}\right)\hat{p}$$

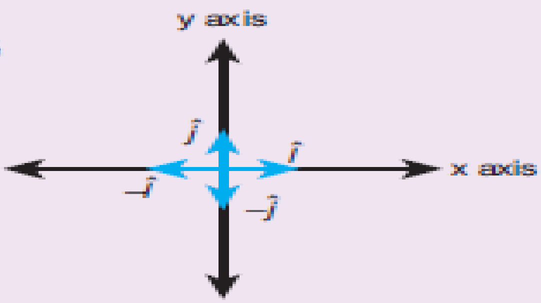
$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\left(\frac{4aq}{r^3}\right)\hat{p} \quad (r >> a)$$

since $2aq\,\hat{p} = \vec{p}$

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3} \quad (r >> a)$$

Magnetic field at a point along the equatorial line due to magnetic dipole





Results from electrostatics

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{(r^2+a^2)^{\frac{3}{2}}}$$

since $\vec{p} = 2qa\hat{p}$

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_\circ}\frac{\vec{p}}{r^3} \qquad (r >> a)$$

$$\vec{B}_{equatorial} = -\frac{\mu_\circ}{4\pi}\frac{p_m}{(r^2+l^2)^{\frac{3}{2}}}\hat{i}$$

$$\left|\vec{p}_m\right| = p_m = q_m \cdot 2l$$

$$\vec{B}_{equatorial} = -\frac{\mu_\circ}{4\pi}\frac{\vec{p}_m}{r^3}$$

Note that magnitude of B_{axial} is twice that of magnitude of $B_{equatorial}$ and the direction of B_{axial} and $B_{equatorial}$ are opposite.

EXAMPLE 3.6

A short bar magnet has a magnetic moment of 0.5 J T^{-1}. Calculate magnitude and direction of the magnetic field produced by the bar magnet which is kept at a distance of 0.1 m from the center of the bar magnet along (a) axial line of the bar magnet and (b) normal bisector of the bar magnet.

Solution

Given magnetic moment 0.5 J T^{-1} and distance r = 0.1 m

(a) When the point lies on the axial line of the bar magnet, the magnetic field for short magnet is given by

$$\vec{B}_{axial} = \frac{\mu_\circ}{4\pi}\left(\frac{2p_m}{r^3}\right)\hat{i}$$

$$\vec{B}_{axial} = 10^{-7}\times\left(\frac{2\times0.5}{(0.1)^3}\right)\hat{i} = 1\times10^{-4}T\ \hat{i}$$

Hence, the magnitude of the magnetic field along axial is $B_{axial} = 1 \times 10^{-4}$ T and direction is towards South to North.

(b) When the point lies on the normal bisector (equatorial) line of the bar magnet, the magnetic field for short magnet is given by

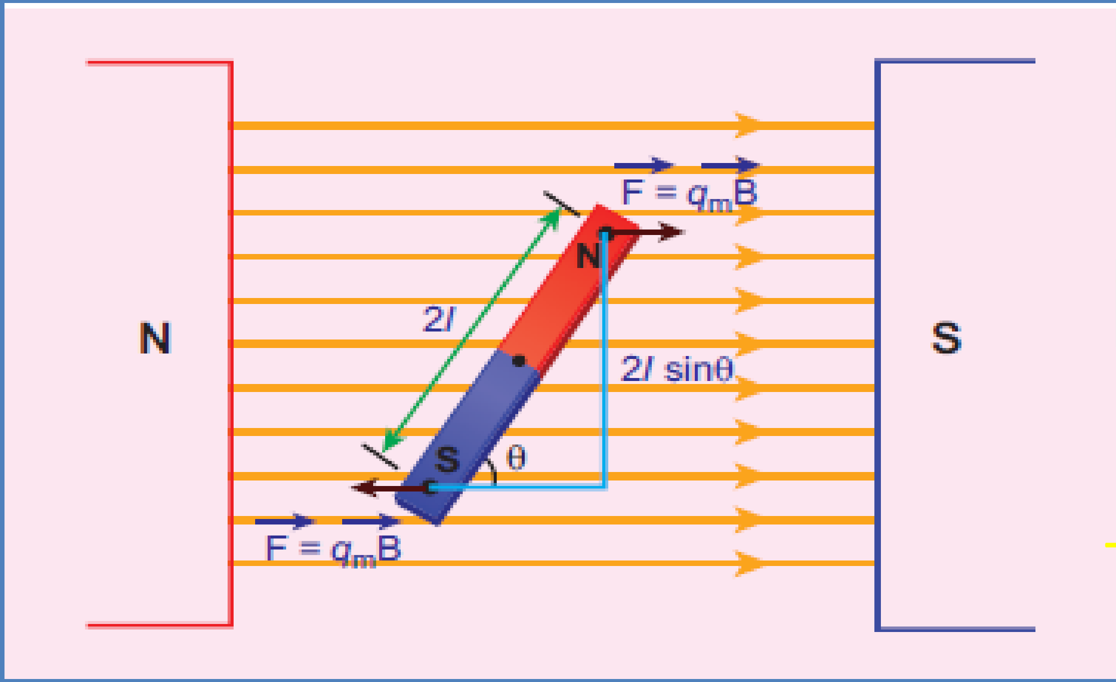
$$\vec{B}_{equatorial} = -\frac{\mu_\circ}{4\pi}\frac{p_m}{r^3}\hat{i}$$

$$\vec{B}_{equatorial} = -10^{-7}\left(\frac{0.5}{(0.1)^3}\right)\hat{i} = -0.5\times10^{-4}T\ \hat{i}$$

Hence, the magnitude of the magnetic field along axial is $B_{equatorial} = 0.5 \times 10^{-4}$ T and direction is towards North to South.

Note that magnitude of B_{axial} is twice that of magnitude of $B_{equatorial}$ and the direction of B_{axial} and $B_{equatorial}$ are opposite.

Magnetic dipole in a uniform magnetic field



The force experienced by north pole

$$\vec{F}_N = q_m \vec{B}$$

The force experienced by south pole

$$\vec{F}_S = -q_m \vec{B}$$

The net force on the dipole

$$\vec{F} = \vec{F}_N + \vec{F}_S = \vec{0}$$

This implies, that the net force acting on the dipole is zero, but forms a couple which tends to rotate the bar magnet clockwise (here) in order to align it along $\vec{B}$.

The moment of force or torque experienced by north and south pole about point O is

$$\vec{\tau} = \overrightarrow{ON} \times \vec{F}_N + \overrightarrow{OS} \times \vec{F}_S$$

$$\vec{\tau} = \overrightarrow{ON} \times q_m \vec{B} + \overrightarrow{OS} \times \left(-q_m \vec{B}\right)$$

By using right hand cork screw rule, we conclude that the total torque is pointing into the paper. Since the magnitudes $\left|\overrightarrow{ON}\right| = \left|\overrightarrow{OS}\right| = l \text{ and } \left|q_m \vec{B}\right| = \left|-q_m \vec{B}\right|$, the magnitude of total torque about point O

$$\tau = l \times q_m B \sin\theta + l \times q_m B \sin\theta$$

$$\tau = 2l \times q_m B \sin\theta$$

$$\tau = p_m B \sin\theta \qquad (\therefore q_m \times 2l = p_m)$$

In vector notation, $\vec{\tau} = \vec{p}_m \times \vec{B}$

DO YOU KNOW?

(a) Why a freely suspended bar magnet in your laboratory experiences only torque (rotational motion) but not any translatory motion even though Earth has non-uniform magnetic field?

It is because Earth's magnetic field is locally(physics laboratory)uniform.

(b) Suppose we keep a freely suspended bar magnet in a non-uniform magnetic field. What will happen?

It will undergo translatory motion (net force) and rotational motion (torque).

EXAMPLE 3.7

Show the time period of oscillation when a bar magnet is kept in a uniform magnetic field is $T = 2\pi\sqrt{\frac{1}{p_m B}}$ in second, where I represents moment of inertia of the bar magnet, p_m is the magnetic moment and is the magnetic field.

Solution

The magnitude of deflecting torque (the torque which makes the object rotate) acting on the bar magnet which will tend to align the bar magnet parallel to the direction of the uniform magnetic field $\vec{B}$ is

$$|\vec{\tau}| = p_m B \sin\theta$$

The magnitude of restoring torque acting on the bar magnet can be written as

$$|\vec{\tau}| = I\frac{d^2\theta}{dt^2}$$

Under equilibrium conditions, both magnitude of deflecting torque and restoring torque will be equal but act in the opposite directions, which means

$$I\frac{d^2\theta}{dt^2} = -p_m B \sin\theta$$

The negative sign implies that both are in opposite directions. The above equation can be written as

$$\frac{d^2\theta}{dt^2} = -\frac{p_m B}{I}\sin\theta$$

This is non-linear second order homogeneous differential equation. In order to make it linear, we use small angle approximation as we did in XI volume II (Unit 10 – oscillations, Refer section 10.4.4) i.e., $\sin\theta \approx \theta$, we get

$$\frac{d^2\theta}{dt^2} = -\frac{p_m B}{I}\theta$$

This linear second order homogeneous differential equation is a Simple Harmonic differential equation. Therefore,

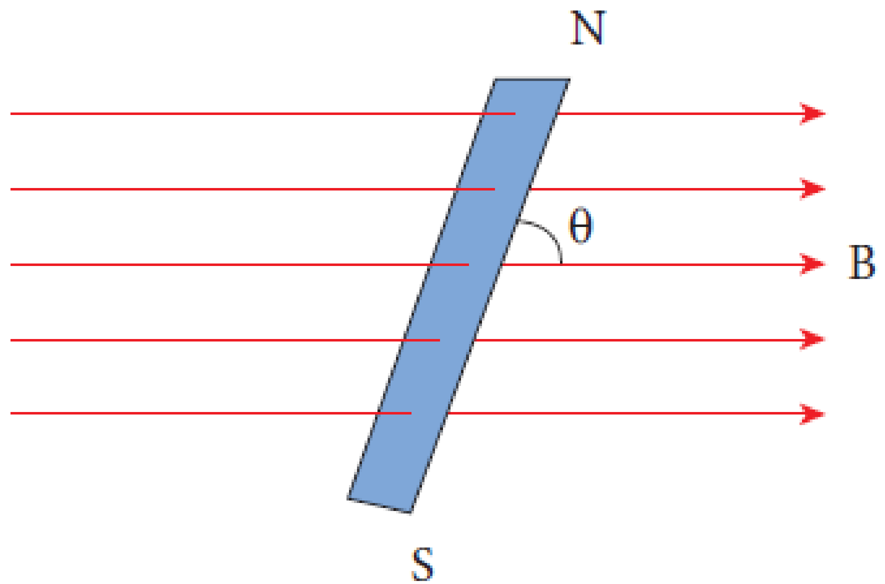
$$\omega^2 = \frac{p_m B}{I} \Rightarrow \omega = \sqrt{\frac{p_m B}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{p_m B}}$$

$$T = 2\pi\sqrt{\frac{I}{p_m B_H}} \text{ in second}$$

where, B_H is the horizontal component of Earth's magnetic field.

Potential energy of bar magnet in uniform magnetic field



When a bar magnet (magnetic dipole) of dipole moment $\vec{p}_m$ is held at an angle θ with the direction of a uniform magnetic field $\vec{B}$,

$$\left|\vec{\tau}_B\right| = \left|\vec{p}_m\right|\left|\vec{B}\right|\sin\theta$$

If the dipole is rotated through a very small angular displacement dθ against the torque τ_B at constant angular velocity, then the work done by external torque $(\vec{\tau}_{ext})$ for this small angular displacement is given by

$$dW = \left|\vec{\tau}_{ext}\right| d\theta$$

The bar magnet has to be moved at constant angular velocity, which implies that $\left|\vec{\tau}_B\right| = \left|\vec{\tau}_{ext}\right|$

$$dW = p_m B \sin\theta \, d\theta$$

Total work done in rotating the dipole from θ' to θ is

$$W = \int_{\theta'}^{\theta} \tau d\theta = \int_{\theta'}^{\theta} p_m B \sin\theta d\theta = p_m B\left[-\cos\theta \, d\theta\right]_{\theta'}^{\theta}$$

$$W = -p_m B(\cos\theta - \cos\theta')$$

This work done is stored as potential energy in bar magnet at an angle θ when it is rotated from θ' to θ and it can be written as

$$U = -p_m B(\cos\theta - \cos\theta')$$

The difference in potential energy between the angular positions θ' and θ.

We can choose the reference point $\theta' = 90^{\circ}$, so that second term in the equation becomes zero and the potential energy is

$$U = -p_m B\,(\cos\theta)$$

The potential energy stored in a bar magnet in a uniform magnetic field is given by

$$U = -\vec{p}_m \bullet \vec{B}$$

Case 1

(i) If $\theta = 0^{\circ}$, then

$$U = -p_m B\,(\cos 0^{\circ}) = -p_m\,\mathrm{B}$$

(ii) If $\theta = 180^{\circ}$, then

$$U = -p_m B\,(\cos 180^{\circ}) = p_m\,\mathrm{B}$$

We can infer from the above two results, the potential energy of the bar magnet is minimum when it is aligned along the external magnetic field and maximum when the bar magnet is aligned anti-parallel to external magnetic field.

EXAMPLE 3.8

Consider a magnetic dipole which on switching ON external magnetic field orient only in two possible ways i.e., one along the direction of the magnetic field (parallel to the field) and another anti-parallel to magnetic field. Compute the energy for the possible orientation. Sketch the graph.

Solution

Let $\vec{p}_m$ be the dipole and before switching ON the external magnetic field, there is no orientation. Therefore, the energy U = 0.

As soon as external magnetic field is switched ON, the magnetic dipole orient parallel ($\theta = 0^{o}$) to the magnetic field with energy,

$$U_{parallel} = U_{minimum} = -p_m B\cos 0$$

$$U_{parallel} = -p_m B$$

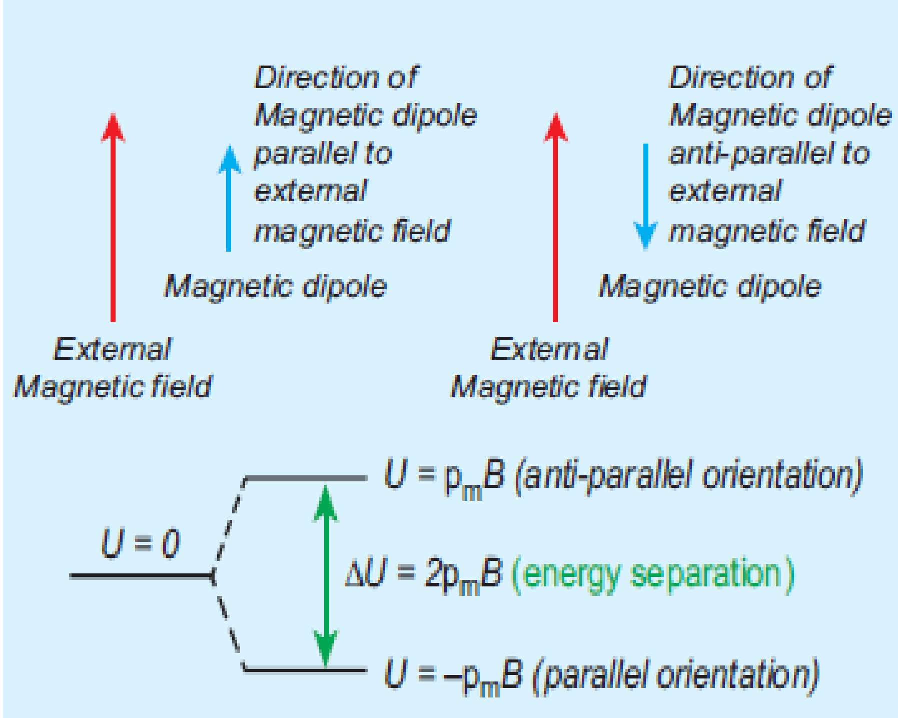
since $\cos 0^{o} = 1$

Otherwise, the magnetic dipole orients anti-parallel ($\theta = 180^{o}$) to the magnetic field with energy,

$$U_{anti-parallel} = U_{maximum} = -p_m B\cos 180$$

$$\Rightarrow U_{anti-parallel} = p_m B$$

since $\cos 180^{o} = -1$



Tangent Galvanometer

- Tangent Galvanometer is a device used to measure very small currents. It is a moving magnet type galvanometer. Its working is based on tangent law.

Tangent law

When a magnetic needle or magnet is freely suspended in two mutually perpendicular uniform magnetic fields, it will come to rest in the direction of the resultant of the two fields.

where B is the magnetic field produced by passing current through the coil of the tangent galvanometer and B_H be the horizontal component of Earth's magnetic field. The action of two magnetic fields, the needle comes to rest making angle θ with B_H

$$B = B_H \tan\theta$$

When an electric current is passed through a circular coil of radius R having N turns, the magnitude of magnetic field at the center is

$$B = \mu_\circ \frac{NI}{2R}$$

The horizontal component of Earth's magnetic field can be determined as

$$B_H = \mu_\circ \frac{NI}{2R} \frac{1}{\tan\theta} \text{ in tesla}$$

Note

1. The current in circuit can be calculated from I = K tan θ, where K is called reduction factor of tangent Galvanometer, where

$$K = \frac{2RB_H}{\mu_o N}$$

2. Sensitivity measures the change in the deflection produced by a unit current, mathematically

$$\frac{d\theta}{dI} = \frac{1}{K\left(1+\frac{I^2}{K^2}\right)}$$

3. The tangent Galvanometer is most sensitive at a deflection of 45°. Generally the deflection is taken between 30° and 60°.

EXAMPLE 3.9

A coil of a tangent galvanometer of diametre 0.24 m has 100 turns. If the horizonta component of Earth's magnetic field is 25×10^{-6} T then, calculate the curren which gives a deflection of 60°.

Solution

The diameter of the coil is 0.24 m Therefore, radius of the coil is 0.12 m.

Number of turns is 100 turns.

Earth's magnetic field is 25×10^{-6} T

Deflection is

$$\theta = 60^\circ \Rightarrow \tan 60^\circ = \sqrt{3} = 1.732$$

$$I = \frac{2RB_H}{\mu_o N}\tan\theta$$

$$= \frac{2\times 0.12\times 25\times 10^{-6}}{4\times 10^{-7}\times 3.14\times 100}\times 1.732 = 0.82\times 10^{-1}A.$$

$$I = 0.082\ A$$